ORGANISE

3 Algebraic vocabulary and notation

Algebraic vocabulary

Algebra is used to represent missing information.

An algebraic x is written using two back-to-back 'c's.

Vocabulary	Examples
A letter can be used to represent any unknown number. This is called a **variable**.	x or y
A number that multiplies a variable is called a **coefficient**.	$3x$ or $2y$
A **constant** is a number that does not change in value.	5 or −8
A **term** is a single variable or a single constant or variables and constants that have been multiplied together.	a $3b$ 12
An **expression** is two or more terms joined by an operator (e.g. +, −, ×). In an expression that is a product, the terms that are being multiplied are called **factors**.	$x - 4 + 3y$ In the expression $7 \times a$, 7 and a are factors.
An **equation** shows that two expressions are equal (indicated by the = sign).	$7y = 21$
A **formula** is a variable that is equal to an expression which uses a different variable.	$s = \frac{d}{t}$

In the expression $2x + 11 - 8y$, notice that:

- the **variables** are x and y
- the **coefficients** are 2 and −8
- the **constant** is 11
- the **terms** are $2x$, 11 and $-8y$

When a variable and a number are multiplied together, the number is always written first. Both $z \times 5$ and $5 \times z$ are written as $5z$.

A product is the result of a multiplication.

Algebraic notation and function machines

Rules of algebraic notation	Examples
When multiplying in algebra, the × sign is not written.	$x \times y$ is written as xy $6 \times (t + 1)$ is written as $6(t + 1)$
When dividing in algebra, the division is usually expressed as a fraction.	$x \div y$ is written as $\frac{x}{y}$ $a \div 3$ is written as $\frac{a}{3}$
When multiplying a variable by 1, the number 1 is not written as the coefficient.	$1 \times w$ is written as w
When multiplying a variable by 0, the number 0 is not written as the coefficient.	$0 \times w$ is written as 0, not $0w$
When multiplying a variable by itself, powers are used.	$y \times y$ is written as y^2 $y \times y \times y$ is written as y^3

A **function machine** applies operations to a variable, called the **input**. The result is called the **output**.

Write the output of the function machine as an **expression**.

b → Subtract 2 → Divide by 5 → Output

b -2 $\div 5$ Output: $\frac{b-2}{5}$

Write the instructions in the function machine as an **equation**.

x → Multiply by 6 → Subtract 3 → 15

x $\times 6$ -3 15

$6x - 3 = 15$

Write the instructions in the function machine as a formula linking b and a.

a → Square it → Add 5 → Divide by 2 → b

a a^2 $+5$ $\div 2$ b

$b = \frac{a^2 + 5}{2}$

Squaring a variable means to multiply the variable by itself, e.g. "a squared" = a^2

Cubing a variable means to multiply the variable by itself and by itself again, e.g. "a cubed" = a^3

RETRIEVE

3 Algebraic vocabulary and notation

Algebraic vocabulary

1 Identify the variables, coefficients, constants and terms in each expression.

a) $3x + y - 12$ Variables: .... Coefficients: ....

Constants: .... Terms: ....

b) $-6f + g + 5$ Variables: .... Coefficients: ....

Constants: .... Terms: ....

c) $10 - de + 4f$ Variables: .... Coefficients: ....

Constants: .... Terms: ....

2 Identify the factors in each expression.

a) $b \times 12$ ....

b) $5 \times \frac{1}{n}$ ....

3 Identify whether each of these is an **expression**, an **equation** or a **formula**.

a) $5a + 4 = 20$ ....

b) $u = x + 2xy - 4$ ....

c) $7e + fg - 18$ ....

Algebraic notation and function machines

4 Write the expressions algebraically.

a) $f \times 3$ .... **b)** $w \times x \times y$ .... **c)** $8 \times (c - d)$ ....

d) $m \div n$ .... **e)** $7 \div a$ .... **f)** $b \div 9$ ....

g) $1 \times d$ .... **h)** $h \times 0$ .... **i)** $n \times n$ ....

5 **a)** Write the instructions in the function machine as an **expression**.

d → **Multiply by 6** → **Add 1** → **Output** ....

b) Write the instructions in the function machine as an **equation**.

y → **Divide by 4** → **Add 3** → **28** ....

c) Write the instructions in the function machine as a **formula** linking b and a.

a → **Subtract 9** → **Square it** → **Multiply by 2** → b ....

ORGANISE

7 Finding parts and wholes from ratios

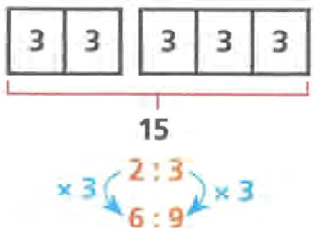
Finding parts of ratios given the whole

Ratios are used to share quantities in different **proportions**.

> You can draw a bar model to help visualise a ratio problem.

There are 15 sweets in a bag. The ratio of red sweets to yellow sweets is 2 : 3.

How many red sweets are in the bag?



$2 + 3 = 5$

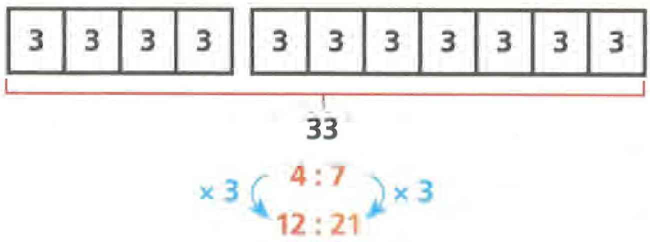
$15 \div 5 = 3$

Multiply both parts of the ratio by 3.

There are 6 red sweets.

A drawer contains white shirts and black shirts only. The number of white shirts to the number of black shirts is in the ratio 4 : 7. There are 33 shirts in total.

How many more black shirts are there than white shirts?



$4 + 7 = 11$

$33 \div 11 = 3$

Multiply both parts of the ratio by 3.

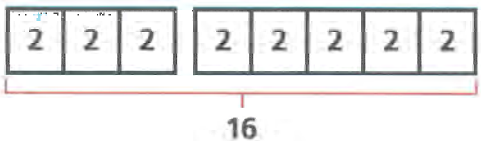
There are 12 white shirts and 21 black shirts.

$21 - 12 = 9$

There are 9 more black shirts than white shirts.

Finding the whole given a part of a ratio

In a fruit bowl, the number of apples to the number of pineapples is in the ratio 3 : 5. There are 6 apples in the bowl. Find the total number of apples and pineapples in the fruit bowl.



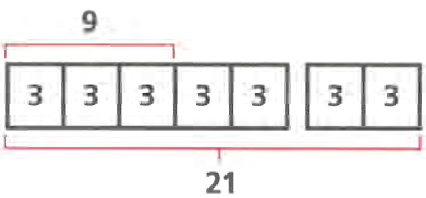
6 apples ÷ 3 parts = 2

3 parts + 5 parts = 8 parts

$2 \times 8 = 16$

There is a total of 16 apples and pineapples in the fruit bowl.

The ratio of red cars to blue cars in a car park is 5 : 2. There are 9 more red cars than blue cars. How many cars are there in total?



Number of parts for red cars is 5.

Number of parts for blue cars is 2.

There are 3 more parts for red cars than for blue cars because $5 - 2 = 3$

3 extra parts represent '9 more' cars.

Sharing 9 cars amongst 3 parts means $9 \div 3$, so each part is worth 3.

There are 7 parts in total because $5 + 2 = 7$

Total number of cars is $3 \times 7 = 21$

RETRIEVE

7 Finding parts and wholes from ratios

Finding parts of ratios given the whole

1. Ravi and Kojo share £30 in the ratio 1 : 4

   How much money does Kojo have?

2. Grace and Kavita share £12 in the ratio 5 : 7

   How much money does Grace have?

3. Share 24 hours in the ratio 4 : 2

4. Divide 110 in the ratio 9 : 2

5. A bag contains 38 counters coloured either orange or white.
   The ratio of orange counters to white counters is 7 : 12

   How many more white counters are there than orange counters?

6. Water and milk are mixed in the ratio 3 : 1 to make a cup of tea.
   300 ml of liquid is mixed altogether.

   How many more millilitres of water are there than milk?

Finding the whole given a part of a ratio

7. Purple paint is made by mixing blue and red paint in the ratio 2 : 3
   There are 6 litres of blue paint.

   How many litres of paint are there altogether?

8. The ratio of slices of lemon to lime in a tub is 5 : 7
   There are 120 slices of lemon in the tub.

   How many slices of lemon and lime are there altogether?

9. A pet shop sells dogs and cats.
   The ratio of dogs to cats is 3 : 4
   There are 18 dogs.

   How many dogs and cats are there altogether?

10. The ratio of red roses to white roses in a flower bed is 3 : 5
    There are 10 more white roses than red.

    How many roses are there altogether?

ORGANISE

7 Introducing ratio

Ratio notation

Ratios are used to compare quantities.

: means 'to' in ratio.

B B B B B Y Y Y

The ratio of black counters to yellow counters is 5 : 3

This means that for every 5 black counters, there are 3 yellow counters.

The ratio of yellow counters to black counters is 3 : 5

This means that for every 3 yellow counters, there are 5 black counters.

The part that is written first is the quantity that should be written first in the ratio.

There are 4 butterflies and 2 snails.

The ratio of butterflies to snails is 4 : 2

The ratio of snails to butterflies is 2 : 4

Ratios can have more than two parts.

G G G G B B B R R

The ratio of green counters to blue counters to red counters is 4 : 3 : 2

Writing ratios

In one month, there were 12 sunny days and 18 rainy days.
The ratio of sunny days to rainy days was 12 : 18
The ratio of rainy days to sunny days was 18 : 12

When ratios have common factors, they can be simplified by dividing both numbers by the highest common factor. For example, 12 : 18 could be simplified to 2 : 3 by dividing both sides by 6. Page 58 covers this in more detail.

Ratios and fractions

Ratios can be expressed as fractions and fractions can be expressed as ratios.

Ratios compare quantities

Quantity ← $\square : \square$ → Quantity

Fractions compare part of a quantity to the whole

$\dfrac{\square}{\square}$ ← Quantity / ← Total amount

Troy and Anji share some money in the ratio 3 : 4. What fraction of the money does Troy get?

Number of parts that Troy gets is 3
Total number of parts is $3 + 4 = 7$
Fraction of money that Troy gets is $\frac{3}{7}$

To find a missing part in a ratio from a fraction, subtract the quantity given in the numerator of the fraction from the total amount in the denominator.

The marbles in a bag are blue, red, and yellow. They are in the ratio 1 : 2 : 5. What fraction of the marbles are red?

Number of parts that are red is 2
Total number of parts is $1 + 2 + 5 = 8$
Fraction of marbles that are red is $\frac{2}{8} = \frac{1}{4}$

A shopping bag contains some fruits and vegetables. The fraction of vegetables in the bag is $\frac{4}{9}$. Find the ratio of fruits to vegetables.

Number of parts that are vegetables is 4
Number of parts that are fruits is $9 - 4 = 5$
Ratio of fruits to vegetables is 5 : 4

RETRIEVE

7 Introducing ratio

Ratio notation

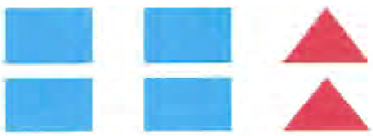
1. Write the number of yellow counters to blue counters as a ratio.

....

2. Write the number of red cars to blue cars as a ratio.

....

3. Write the number of rectangles to triangles as a ratio.



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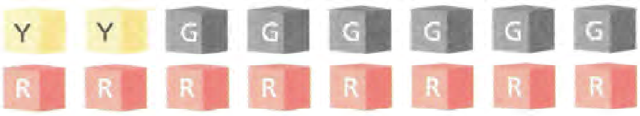
4. Write the number of red cubes to yellow cubes to grey cubes as a ratio.



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Writing ratios

5. There are 30 chairs and 15 tables in a classroom.

Write the ratio of chairs to tables. ....

6. A bag contains 8 orange sweets and 15 red sweets.

a) Write the ratio of orange sweets to red sweets. ....

b) Write the ratio of red sweets to orange sweets. ....

7. A bus is carrying 22 seated passengers and 16 standing passengers.

a) Write the ratio of seated passengers to standing passengers. ....

b) Write the ratio of standing passengers to seated passengers. ....

8. A team plays 24 matches. 10 are wins, 6 are draws and 8 are defeats.

Write the ratio of wins to draws to defeats. ....

Ratios and fractions

9. A bag has black and yellow counters in the ratio 7 : 4

What fraction of the counters are yellow? ....

10. Pancake mix is made by mixing milk, flour, and eggs in the ratio 3 : 3 : 1

What fraction of the pancake mix is flour? ....

11. Viv bought some jumpers and shirts. $\frac{1}{9}$ of his shopping was jumpers.

Write the ratio of jumpers to shirts. ....

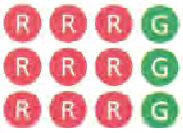
7 Simplifying ratios

Simplifying ratios expressed as integers

Ratios can be simplified by dividing all the numbers in the ratio by a **common factor** until they cannot be divided any more. To do this in one step, divide both numbers by the **highest common factor**. Simplified ratios should have integers, not decimals.

A factor is a number that divides exactly into another number without a remainder.

What is the ratio of red counters to green counters? Write the ratio in its simplest form.



$$= 9 : 3 \quad (\div 3)$$
$$= 3 : 1$$

Ratio of red to green is 3 : 1

Simplifying a ratio that has three parts is the same as simplifying a ratio that has two parts. Find a **common factor** of each number in the ratio and simplify.

Write the ratio 6 : 3 : 12 in its simplest form.

$$6 : 3 : 12 \quad (\div 3)$$
$$2 : 1 : 4$$

The highest common factor of 6, 3 and 12 is 3, so each number is divided by 3 to write the ratio in its simplest form.

6 : 3 : 12 in its simplest form is 2 : 1 : 4

Simplifying ratios in different units

To simplify ratios that are in different units, convert both numbers into the smaller unit, then simplify by dividing by the **highest common factor**.

Write the ratio 5 km : 2 m in its simplest form.

$$5\,\text{km} : 2\,\text{m}$$
$$5000\,\text{m} : 2\,\text{m} \quad (\div 2)$$
$$2500\,\text{m} : 1\,\text{m}$$

Change both sides to metres. There are 1000 m in 1 km so multiply 5 km by 1000.

5 km : 2 m in its simplest form is 2500 : 1

Simplifying ratios expressed as decimals

When a ratio is expressed using decimals, simplify it by writing the ratio using whole numbers.

Write the ratio 0.5 : 2 in its simplest form.

$$0.5 : 2 \quad (\times 10)$$
$$5 : 20 \quad (\div 5)$$
$$1 : 4$$

Multiply both parts of the ratio by 10 to convert the decimal to a whole number, then simplify by dividing by the highest common factor.

0.5 : 2 in its simplest form is 1 : 4

Simplifying ratios into their unitary form

The unitary form of a ratio is 1 : n

Write the ratio 4 : 5 in its unitary form.

$$4 : 5 \quad (\div 4)$$
$$1 : 1.25$$

4 : 5 in unitary form is 1 : 1.25

Simplifying ratios expressed as fractions

When a ratio is expressed using fractions, convert the fractions in the ratio so that they have a **common denominator**. Then multiply both fractions by the **denominator** and simplify using the **highest common factor**.

Write the ratio $\frac{5}{6} : \frac{1}{2}$ in its simplest form.

$$\frac{5}{6} : \frac{1}{2}$$
$$\frac{5}{6} : \frac{3}{6} \quad (\times 6)$$
$$5 : 3$$

To convert $\frac{1}{2}$ into a fraction with a denominator of 6, multiply the numerator and denominator by 3.

$\frac{5}{6} : \frac{1}{2}$ in its simplest form is 5 : 3

RETRIEVE

7 Simplifying ratios

Simplifying ratios expressed as integers

1. Express each ratio in its simplest form.

a) 14 : 21 ....

b) 18 : 14 ....

c) 60 : 20 : 50 ....

d) 12 : 30 : 24 ....

Simplifying ratios in different units

2. Express each ratio in its simplest form.

a) 2 km : 4 m ....

b) 6 litres : 3 millilitres ....

c) 50 mm : 12 cm ....

d) 9 kilograms : 80 grams ....

Simplifying ratios into their unitary form

3. Express each ratio in its unitary form.

a) 3 : 9 ....

b) 6 : 24 ....

c) 2 : 5 ....

d) 10 : 8 ....

Simplifying ratios expressed as decimals

4. Express each ratio in its simplest form.

a) 0.2 : 3 ....

b) 0.25 : 4 ....

c) 1 : 3.5 ....

d) 1.6 : 2.8 ....

Simplifying ratios expressed as fractions

5. Express each ratio in its simplest form.

a) $\frac{5}{6} : \frac{1}{6}$ ....

b) $\frac{7}{8} : \frac{3}{4}$ ....

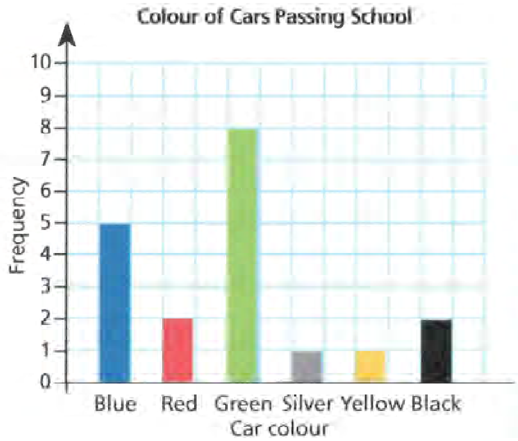
c) $\frac{4}{6} : \frac{3}{8}$ ....

d) $\frac{3}{7} : 2$ ....

Interpreting statistical representations

Interpreting statistical measures and representations

The bar chart shows information about the colour of cars passing a school over a 10-minute period.



a) How many blue cars drove past?

Read off the frequency from the y-axis of the blue car category.

5 blue cars drove past.

b) How many more green cars than silver cars drove past?

Find how many cars of each colour drove past and find the difference between them by **subtracting**.

7 more green cars than silver cars drove past.

c) What colour of car drove past the most?

Green cars had the highest frequency, so green cars drove past most often. This is the **mode**.

d) How many cars drove past in total?

19 cars drove past in total. To find the total, add together the frequencies of each colour of car.

Choosing an appropriate statistical measure

When choosing an appropriate statistical measure, consider the advantages and disadvantages of each.

Measure of central tendency	Advantage	Disadvantage
Mean	Takes account of every value	Affected by very large or very small values
Median	Unaffected by very large or very small values	May not be an actual number in the data set
Mode	Only average that can be used for qualitative data	There may be more than one mode or no mode

The salaries of five employees in a company are:

£23000 £25000 £30000 £33000 £120000

Which statistical measure should be used to represent the average?

The **median** should be used because it is unaffected by very large or very small values. £120 000 is a very large value compared to the others.

A shop wants to find the average size of shoe sold to help it to decide which size it needs most stock of. The sizes sold on a particular day are:

3, 4, 4, 5, 6, 7, 8, 8, 9, 9, 9, 9, 9, 10, 10, 11, 12, 14

Which statistical measure should be used for the average?

The **mode** (9) should be used because it shows which shoe size is in greatest demand.

The heights of some Year 8 students, in metres, are:

1.72, 1.54, 1.57, 1.50, 1.55, 1.46, 1.63, 1.61

Which statistical measure should be used for the average height?

The **mean** should be used because it takes account of every value and there are few very large or very small values in the data set.

The range is not a measure of central tendency. It measures the spread of the data set.

The daily temperatures across March last year for two cities are summarised in this table.

City	Mean maximum daily temperature	Range of maximum daily temperature
A	22°C	6°C
B	22°C	13°C

Which city should you choose if you want to enjoy high temperatures? Justify your answer.

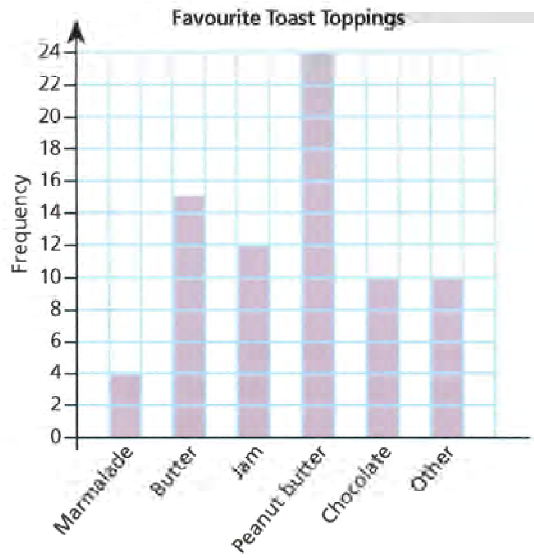
City A should be the city you choose to visit. Both cities have the same mean, but city A has the smaller range. This means that the temperature is more consistently high in city A compared to city B.

RETRIEVE 6

Interpreting statistical representations

Interpreting statistical measures and representations

1 The bar chart shows the preferred toast toppings of a group of students.



a) How many students prefer chocolate on their toast? ....

b) How many students prefer marmalade on their toast? ....

c) How many more students prefer butter on their toast than jam? ....

d) What is the mode of this data? ....

e) How many students took part in the survey? ....

Choosing an appropriate statistical measure

2 A boutique had daily sales of **£326, £540, £385, £450, £2435, £459** and **£493** over the last week.

Is the mean or median a more reliable measure of central tendency? Justify your answer.

....

....

3 The favourite subjects of some students were collected and recorded:

French, PE, Maths, Science, Maths, ICT, Maths, DT, Maths

Which measure of central tendency can best be used to describe this data? Justify your answer.

....

....

4 The daily temperatures across March last year for two cities are summarised in this table.

City	Mean maximum daily temperature	Range of maximum daily temperature
C	12°C	8°C
D	21°C	8°C

Which city should you choose to visit if you want to enjoy high temperatures? Justify your answer.

....

....

ORGANISE

6 Pie charts and scatter graphs

Pie charts

Pie charts use a circle to display data. Pie charts show the **proportion** or **fraction** of each category within the data compared to the whole circle.

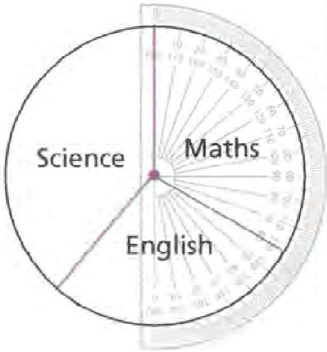
To find the size of each angle, multiply the fraction by 360 because there are 360° in a circle. Always label the sections of a pie chart.

The frequency table (right) shows the favourite subject of 36 students. Draw a pie chart to show this data.

Subject	Frequency
Maths	12
English	10
Science	14

To find the size of each angle, express each subject's frequency as a fraction of the total number of students asked, and multiply this by 360°.

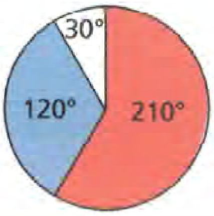
Subject	Frequency	Fraction	Angle
Maths	12	$\frac{12}{36}$	120°
English	10	$\frac{10}{36}$	100°
Science	14	$\frac{14}{36}$	140°



To draw a pie chart, use a **protractor** to mark off the angles that are needed.

You can find the frequency of a category if given the total frequency and the size of an angle in a pie chart.

The pie chart shows the proportion of different fish in a tank. There are 24 fish in total.



Key:
- Blue fish
- White fish
- Red fish

Work out how many blue fish are in the tank.

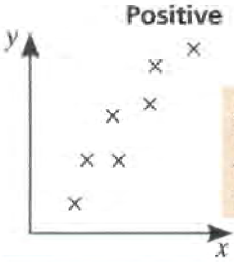
$\frac{120}{360} \times 24 = 8$

There are 8 blue fish in the tank.

Scatter graphs

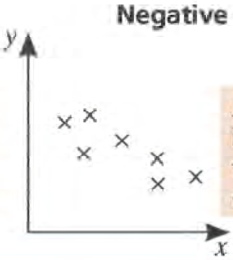
Scatter graphs show the relationship between two sets of **numerical** data.
The relationship is described using **correlation**. There are three types of correlation:

Positive



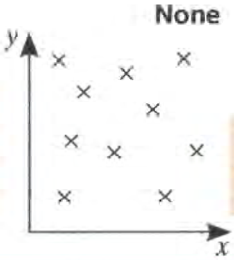
As one increases the other increases

Negative



As one increases the other decreases

None



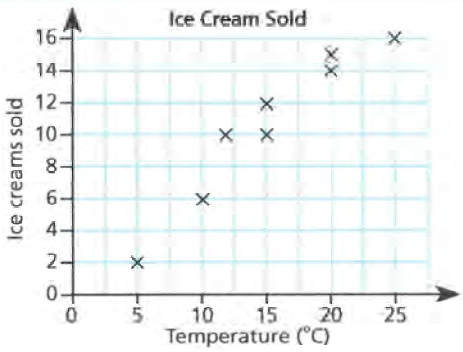
No pattern or relationship

Correlation is strong if the points on a scatter graph closely follow a straight line. Correlation is weak if the points are more loosely spread from a straight line.

A shopkeeper records the temperature for 8 days and the number of ice creams sold.

Temperature (°C)	Ice creams sold
20	14
15	10
25	16
10	6
20	15
5	2
15	12
12	10

a) Plot the data on a scatter graph.



b) What type of correlation does the graph show?

A positive correlation. The higher the temperature, the more ice creams that are sold.

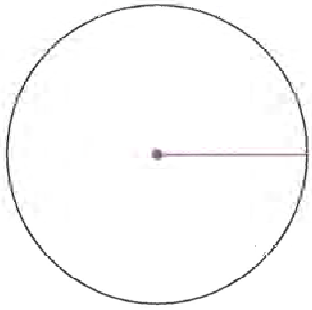
RETRIEVE 6

Pie charts and scatter graphs

Pie charts

1 The frequency table shows the favourite colour of 36 students.

Colour	Frequency	Fraction	Angle
White	9		
Green	20		
Red	7		



a) Complete the table.

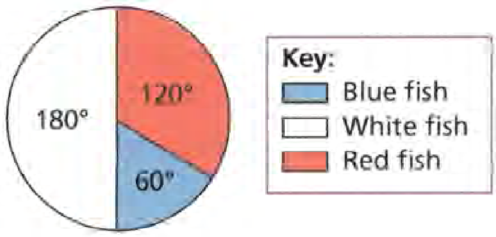
b) Draw a pie chart to represent this information.

2 The pie chart shows the proportion of different fish in a tank. There are 48 fish in total.

a) Work out the number of blue fish in the tank. ....

b) Work out the number of white fish in the tank. ....

c) Work out the number of red fish in the tank. ....

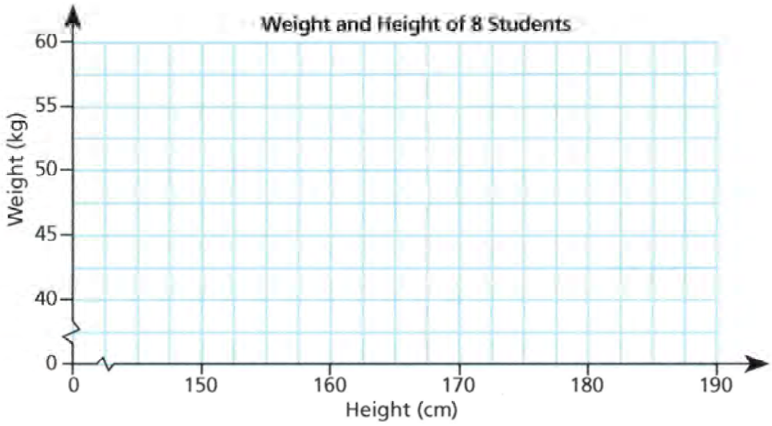


Scatter graphs

3 The height and weight of 8 students are shown in the table.

Height (cm)	Weight (kg)
150	45
156	46
156	51
181	57
170	51
172	58
165	52
180	50

a) Plot the data on a scatter graph.

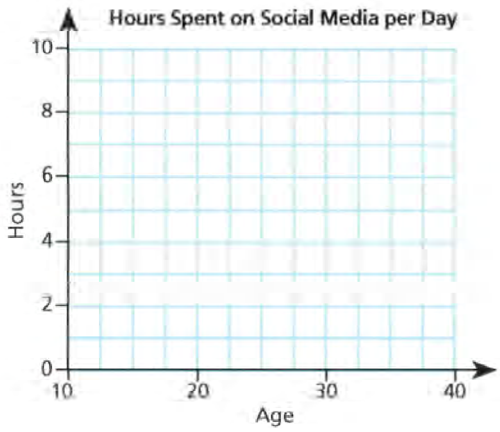


b) Describe the correlation between height and weight.

....

4 The number of hours spent on social media by people of different ages is shown in the table.

a) Plot the data on the scatter graph.



Age	Hours spent on social media per day
10	10
15	8
21	6
27	4
32	2

b) Describe the correlation between age and the number of hours spent on social media in a day.

....

ORGANISE

7 Properties of polygons

Properties of quadrilaterals

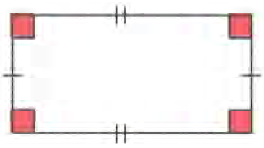
A **polygon** is a closed **2D shape** with sides that are straight lines. A **quadrilateral** is a polygon with:

- four sides
- four vertices
- interior angles that add up to 360°.

A vertex is a corner. This is where two edges meet.

Adjacent means 'next to'.

Rectangle

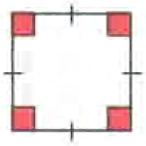


All angles are **equal** (90°).
Opposite sides are **equal** in length.
Opposite sides are **parallel**.

Parallelogram

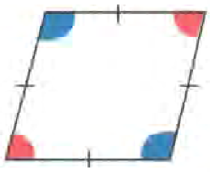
Opposite angles are **equal**.
Opposite sides are **parallel**.
Opposite sides are **equal**.
Diagonals **bisect** each other.

Square



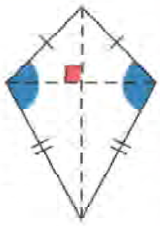
All angles are **equal** (90°).
All sides are **equal** in length.
Opposite sides are **parallel**.

Rhombus



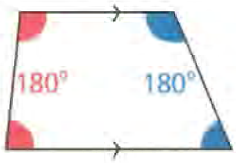
Opposite angles are **equal**.
All sides are **equal** in length.
Opposite sides are **parallel**.

Kite



Two pairs of **adjacent equal sides**.
Opposite angles between the sides of different length are **equal**.
Its long diagonal **bisects** the short diagonal at right angles.

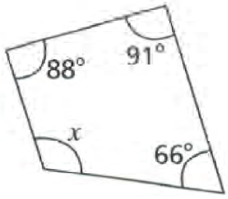
Trapezium



One pair of **parallel** sides.
The two angles at the end of each non-parallel side **add to 180°**.

Work out the size of the angle marked x in this quadrilateral.

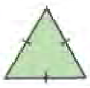
To work out the size of x, add together the angles given and subtract from 360° because interior angles in a quadrilateral sum to 360°.

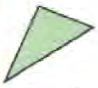
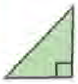
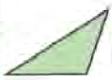


$88° + 91° + 66° = 245°$
$360° - 245° = 115°$
$x = 115°$

Properties of triangles

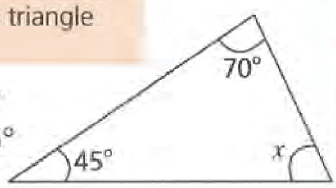
Triangles can be categorised by the length of their sides or by the size of their largest angle.

Categorised by side	
	An **equilateral triangle** has three equal sides
	An **isosceles triangle** has two equal sides
	A **scalene triangle** has no equal sides

Categorised by angle	
	An **acute triangle** has three angles < 90°
	A **right triangle** has one angle of 90°
	An **obtuse triangle** has one angle > 90°

Find the angle marked x in this triangle.

To work out the size of x, add together the angles given and subtract from 180° because interior angles in a triangle sum to 180°.



$45° + 70° = 115°$
$180° - 115° = 65°$
$x = 65°$

RETRIEVE

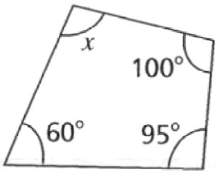
7 Properties of polygons

Properties of quadrilaterals

1 Complete this table of the properties of quadrilaterals.

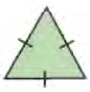
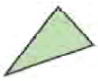
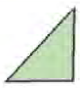
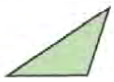
		Properties	
Quadrilateral	**Image**	**Sides**	**Angles**
Square		• Four equal sides • Opposite sides are parallel	
Rectangle			
			• Two opposite pairs of equal angles
Rhombus			• Two opposite pairs of equal angles
		• Two pairs of equal sides	• One opposite pair of equal angles
Trapezium			

2 Find the size of the angle marked x in this quadrilateral.

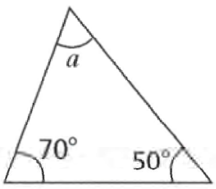


Properties of triangles

3 Complete this table of the properties of triangles.

Triangle	Equilateral triangle		Scalene triangle		Right triangle	Obtuse triangle
Image						
Properties		• Two sides are equal length • Two equal angles		• Has three angles that are all less than 90°	• Has one angle that	• Has one angle that ....

4 Find the size of the angle marked a in this triangle.

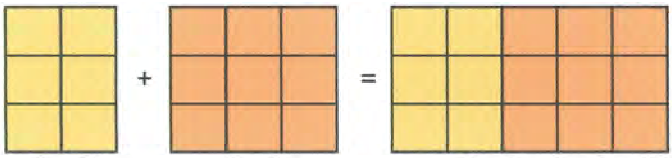


ORGANISE

4 Solving equations with brackets

Expanding brackets using the distributive law

The **distributive law** says that the product of two numbers multiplied together is equal to the product of those numbers split into groups (**partitioned**).



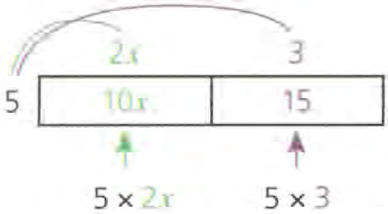
$3 \times 2 \quad + \quad 3 \times 3 \quad = \quad 3 \times 2 + 3 \times 3 = 15$

$3 \times (2 + 3) = 15$

The distributive law can be shown without visual aids by **expanding the bracket**.

$$\begin{aligned} 3(2 + 3) &= (3 \times 2) + (3 \times 3) \\ &= 6 + 9 \\ &= 15 \end{aligned}$$

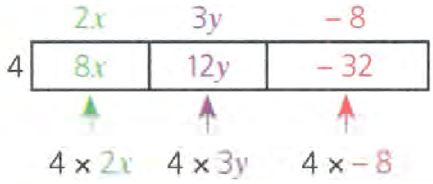
The same rule applies when there are variables inside or outside the brackets. $5(2x + 3)$ can be expanded using an area model. Draw a large rectangle partitioned into smaller rectangles and find the area of each.



$$\begin{aligned} &5(2x + 3) \\ &= (5 \times 2x) + (5 \times 3) \\ &= 10x + 15 \end{aligned}$$

To expand a pair of brackets with more than two terms, you must multiply by each term.

To expand $4(2x + 3y - 8)$, multiply 4 by each term in the bracket.

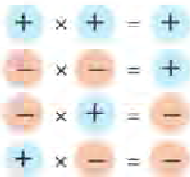


$$\begin{aligned} &4(2x + 3y - 8) \\ &= 8x + 12y - 32 \end{aligned}$$

Multiply what is outside the brackets by everything inside the brackets.

When expanding brackets, remember that:

- multiplying or dividing two negative numbers gives a positive number
- multiplying or dividing a negative and a positive gives a negative number
- adding a negative number is the same as subtracting
- subtracting a negative number is the same as adding.



Expand $3m(2 - 7n)$

Multiply $3m$ by each term inside the bracket and carry the negative sign through to the answer.

$$\begin{aligned} 3m(2 - 7n) &= (3m \times 2) + (3m \times -7n) \\ &= 6m - 21mn \end{aligned}$$

Solving equations with brackets

To solve an equation with one pair of brackets, decide whether it will be easier to expand the brackets first or to divide by the number in front of the brackets.

Solve for k. $3(k - 5) = 6$

$$\begin{aligned} 3 \times k + 3 \times -5 &= 6 \\ 3k - 15 &= 6 \\ +15 \quad &+15 \\ 3k &= 21 \\ \div 3 \quad &\div 3 \\ k &= 7 \end{aligned}$$

$$\begin{aligned} 3(k - 5) &= 6 \\ \div 3 \quad &\div 3 \\ k - 5 &= 2 \\ +5 \quad &+5 \\ k &= 7 \end{aligned}$$

To check, substitute $k = 7$ into $3(k - 5) = 6$

$3(7 - 5) = 3 \times 2 = 6$ ✔

If an equation has more than one pair of brackets, expand all the brackets and simplify before solving.

Solve $3(2j - 4) + 2(j + 3) = 26$

First expand both brackets.

$$\begin{aligned} &3(2j - 4) \\ &= (3 \times 2j) + (3 \times -4) \\ &= 6j - 12 \end{aligned}$$

$$\begin{aligned} &2(j + 3) \\ &= (2 \times j) + (2 \times 3) \\ &= 2j + 6 \end{aligned}$$

$$\begin{aligned} 6j - 12 + 2j + 6 &= 26 \\ 8j - 6 &= 26 \\ +6 \quad &+6 \\ 8j &= 32 \\ \div 8 \quad &\div 8 \\ j &= 4 \end{aligned}$$

RETRIEVE

4 Solving equations with brackets

Expanding brackets using the distributive law

1 Expand each expression and simplify where possible by combining like terms.

a) $3(2k - 4) + 5k$ **b)** $k(2m - 4)$ **c)** $4(2x + 3y - 5)$

....

Solving equations with brackets

2 Solve the following equations.

a) $3(k - 5) = 12$

$k =$ ....

b) $2x + 3(4 - x) = 10$

$x =$ ....

3 Solve the following equations.

a) $2(x + 3) + 4(x - 5) = 10$

$x =$ ....

b) $5(y - 2) = 3(y - 2) + 2$

$y =$ ....

ORGANISE

4

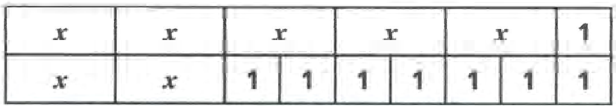
Solving equations with variables on both sides

Using bar models (when all terms are positive)

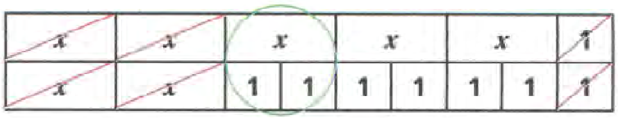
Bar models are a good way to visualise equations with variables on both sides of the equals symbol when all the terms are positive.

A bar model doesn't work as well when there are negative terms. In that case, it is best to use algebra tiles.

This bar model shows the equation $5x + 1 = 2x + 7$:



To solve the equation, 'cancel out' the terms that are the same in both bars.



This leaves $3x = 6$, or $1x = 2$, so the answer is $x = 2$.

Using algebra tiles (when terms are positive and/or negative)

To solve equations with variables on both sides using algebra tiles, carry out inverse operations so that all the variable tiles are on one side of the equation and the constants are on the other.

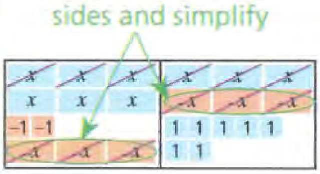
It doesn't matter if the variables are on the left or the right, as long as they are all on the same side.

These algebra tiles show the equation $6x - 2 = 3x + 7$

$6x - 2 = 3x + 7$

Get the variables on one side.

Add 3 $-x$ tiles to both sides and simplify

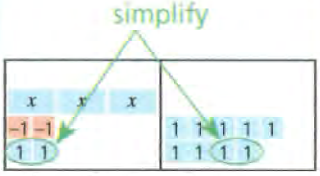


$6x - 2 - 3x = 3x + 7 - 3x$

$3x - 2 = 7$

Isolate the x term then simplify.

Add 2 to both sides and simplify

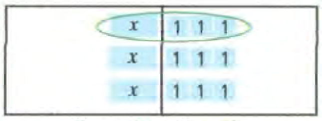


$3x - 2 + 2 = 7 + 2$

$3x = 9$

Divide each side into three equal groups.

1 x tile equals 3 ones tiles



$3x \div 3 = 9 \div 3$

$x = 3$

To check, substitute $x = 3$ into $6x - 2 = 3x + 7$

$6 \times 3 - 2 = 18 - 2 = 16$

$3 \times 3 + 7 = 9 + 7 = 16$ ✔

Solving without visual aids

To solve $2y + 6 = 4y$, use inverse operations to get the y term on one side of the equation. In this case, subtracting $2y$ from both sides is easier because subtracting $4y$ would leave a negative y term, but both methods give the same answer:

$2y + 6 = 4y$

$2y + 6 - 2y = 4y - 2y$ Subtract $2y$ from both sides.

$6 = 2y$ Simplify.

$6 \div 2 = 2y \div 2$ Divide both sides by 2.

$3 = y$

$2y + 6 = 4y$

$2y + 6 - 4y = 4y - 4y$ Subtract $4y$ from both sides.

$-2y + 6 = 0$ Simplify.

$-2y + 6 - 6 = 0 - 6$ Subtract 6 from both sides.

$-2y = -6$ Simplify.

$-2y \div -2 = -6 \div -2$ Divide both sides by -2.

$y = 3$

To check the answer is correct, substitute $y = 3$ into $2y + 6 = 4y$

$2 \times 3 + 6 = 6 + 6 = 12$

$4 \times 3 = 12$ ✔

Similarly, you could subtract 6 from both sides first or divide each term by 2 first and you would arrive at the same answer.

RETRIEVE 4

Solving equations with variables on both sides

Using bar models (when all terms are positive)

1 Draw bar models and solve the following equations.

a) $2x + 8 = 3x + 6$

b) $3b + 5 = 4b + 1$

$x =$ ....

$b =$ ....

Using algebra tiles (when terms are positive and/or negative)

2 Show the equation $4x + 6 = 6x - 4$ using algebra tiles. Then use the tiles or your preferred method to solve the equation. If you do not have algebra tiles to hand, you can cut up bits of paper and label them.

$x =$ ....

Solving without visual aids

3 Solve each of the following equations. Show your working.

a) $2x + 6 = 5x - 15$

$x =$ ....

b) $3x - 4 = 20 - x$

$x =$ ....

ORGANISE

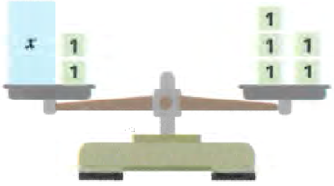
4 Solving one-step equations

Equations and inverse operations

An expression is a collection of **terms** that can be **variables** or **constants** without an equals symbol.

An **equation** consists of one or more **expressions**. Both sides of the equation are equal in value (as shown by the = symbol).

A set of **balance scales** can help to visualise equations. This set of scales represents the equation $x + 2 = 5$.



To solve equations, you need to use **inverse operations** (these 'undo' other operations).

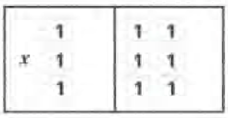
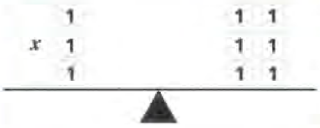
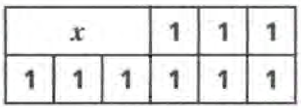
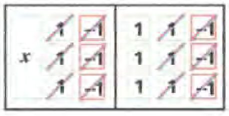
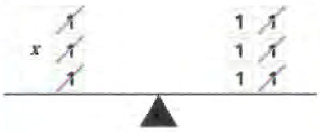
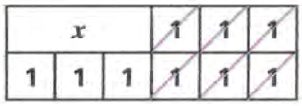
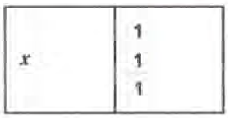
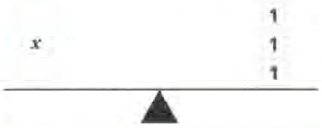
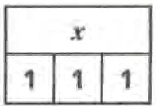
Inverse operations		
Addition +	⇔	Subtraction −
Multiplication ×	⇔	Division ÷
Powers (e.g. x^2, x^3)	⇔	Roots (e.g. $\sqrt{x}$, $\sqrt[3]{x}$)

Using visual aids to solve equations

To '**solve** for x' means to find the value of x. The x term must be **isolated** on one side of the equation. The aim is to get the variables to one side of the equation and the constants to the other.

This equation mat, scales and bar model all show the equation $x + 3 = 6$.

Whatever you do to one side of the equation, you must do to the other to keep it balanced.

Algebraic steps	Equation mat	Scales	Bar model
Lay out the equation. $x + 3 = 6$			
Use inverse operations to isolate x. $x + 3 - 3 = 6 - 3$ Subtract 3 from both sides	Place three −1 tiles on both sides. The +1 and −1 tiles 'cancel out' because they add up to 0. 	Remove 3 ones tiles from both sides. 	'Cancel out' the parts that are the same on both bars. 
Simplify $x = 3$			

Solving without visual aids

To solve an equation without visual aids, use inverses and carry out the same operation on both sides to keep it balanced like a set of scales.

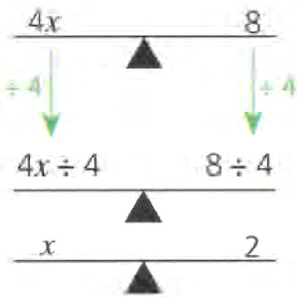
$4x = 8$

The inverse of × 4 is ÷ 4, so divide both sides by 4.

$4x \div 4 = 8 \div 4$

$x = 2$

To check: $4 \times 2 = 8$ ✔

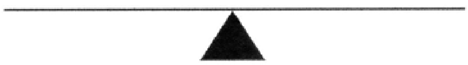


4 Solving one-step equations

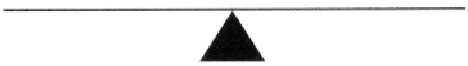
Equations and inverse operations

1 Show each equation on a set of balance scales using algebra tiles.

a) $2x + 4 = 8$



b) $3x + 2 = 11$



Using visual aids to solve equations

2 Show $x + 6 = 8$ using algebra tiles and find the value of x.

$x =$ ....

Solving without visual aids

3 Solve the following equations for x.

a) $x + 9 = 15$

$x =$ ....

b) $x \div 5 = 3$

$x =$ ....

c) $12x = 96$

$x =$ ....

ORGANISE

5 Properties of 3D shapes 1

Definitions

3D shapes are **solid shapes or objects** that have three dimensions: length, width and height.

3D means three-dimensional.

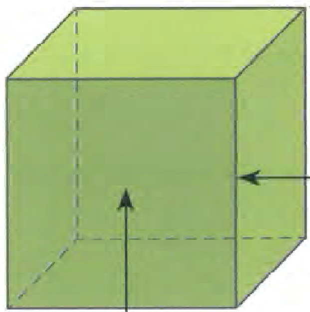
Some real-life examples of 3D shapes are a can, a football and a cereal box.

Cylinder

Sphere

Cuboid

3D shapes have **faces**, **edges** and **vertices**.



A **vertex** is a corner where **edges** meet.

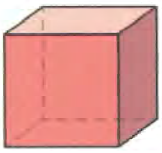
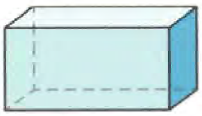
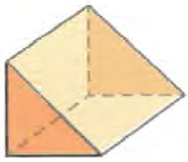
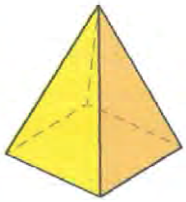
An **edge** is the line where two **faces** join.

A **face** is a flat or curved surface on a 3D shape.

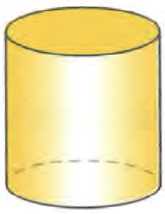
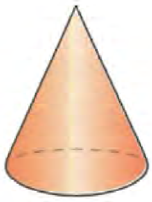
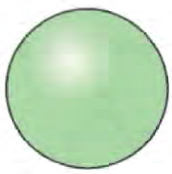
'Vertices' is the plural of 'vertex'.

Properties of 3D shapes

3D shapes with flat faces only

	Cube	Cuboid	Triangular prism	Square-based pyramid
Shape				
Faces	6	6	5	5
Edges	12	12	9	8
Vertices	8	8	6	5

3D shapes with curved faces

	Cylinder	Cone	Sphere
Shape			
Faces	3	2	1
Edges	2	1	0
Vertices	0	1	0

RETRIEVE

5 Properties of 3D shapes 1

Definitions

1. Define what a face of a 3D shape is.

2. Define what an edge of a 3D shape is.

3. Define what a vertex of a 3D shape is.

Properties of 3D shapes

4. Name a 3D shape that has 6 faces, 12 edges and 8 vertices.

5. Name the 3D shape that has 2 faces, 1 edge and 1 vertex.

6. Name the 3D shape that has 5 faces, 8 edges and 5 vertices.

7. Name the 3D shape that has 3 faces, 2 edges and 0 vertices.

8. Name the 3D shape that has 5 faces, 9 edges and 6 vertices.