ORGANISE 6

Measures of central tendency and spread

Measures of central tendency

A measure of central tendency is a single value that attempts to describe a set of data by identifying its central position. **Mean**, **median** and **mode** are measures of central tendency:

- The mean is an **arithmetic average** found by adding together the values and dividing the total by the number of values in the data set.
- The median is the **middle value** when the data is ordered.
- The mode is the **most common** value in the data set.

The number of goals that were scored in 7 football matches are recorded. Find the mean number of goals scored.

1 3 4 6 6 7 8

> To determine the mean, divide the total sum by the total number of items in the data set.

$1 + 3 + 4 + 6 + 6 + 7 + 8 = 35$

$35 \div 7 = 5$

The mean is 5 goals per game.

Find the median of this data set showing the number of goals scored in 7 football matches.

1 6 7 4 6 8 3

> To find the median, put the numbers in **ascending** or **descending** order, and find the **middle value** by crossing out the numbers at each end of the data set.

~~1~~ ~~3~~ ~~4~~ **6** ~~6~~ ~~7~~ ~~8~~

The median is 6 goals.

Find the median of the data set.

9 9 9 10 10 14 15 16 19 20

> To find the median of a data set that has an even number of values, find the two middle numbers, add the two numbers together and divide the total by 2.

~~9~~ ~~9~~ ~~9~~ ~~10~~ **10 14** ~~15~~ ~~16~~ ~~19~~ ~~20~~

$10 + 14 = 24$

$24 \div 2 = 12$

Median = 12

Find the mode of this data set showing the number of goals scored in 7 football matches.

1 3 4 6 6 7 8

> To find the mode, look for the value that occurs the most often (i.e. the value that repeats more than any other value).

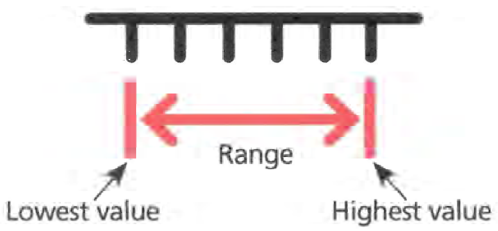
6 is the only value that is repeated in this data set.

The mode is 6 goals.

> It is possible to have more than one mode or no mode at all.

Measuring the spread of data

The range measures the **spread** of the data set. To find the range, work out the difference between the highest and lowest values in the data set.



Find the range of this data set showing the number of goals scored in 7 football matches.

1 3 4 6 6 7 8

Range = highest value – lowest value

Range = 8 – 1

Range = 7 goals

RETRIEVE 6

Measures of central tendency and spread

Measures of central tendency

1 The number of hours that 5 students spent studying in one week was recorded as follows:

10 5 3 5 7

a) Work out the mean.

....

b) Work out the median.

....

c) Work out the mode.

....

2 The shoe sizes of 8 students were recorded as follows:

3 3 5 4 3 4 7 3

a) Work out the mean.

....

b) Work out the median.

....

c) Work out the mode.

....

Measuring the spread of data

3 The number of hours that 5 students spent studying in one week was recorded as follows:

10 5 1 5 7

Work out the range.

....

4 The shoe sizes of 8 students were recorded as follows:

3 3 5 4 3 4 7 3

Work out the range.

....

6 Frequency tables

Data types and frequency tables

Data is a set of information that can be analysed. Data can be collected by using tables, which makes it easier to read and understand.

Qualitative data is data that describes something (e.g. colour of hair; colour of eyes).

Quantitative data is data that is numerical (e.g. height; shoe size).

There are two types of quantitative data:

- **Discrete** data can only take certain values (e.g. the number of people in a room; the number of pages in a book).
- **Continuous** data can take any value (e.g. the time taken to run a race; the mass of an object).

A **grouped** frequency table is used to organise and simplify a large set of data into smaller groups. A frequency table containing **ungrouped** data has data that has not been put into categories or simplified.

The heights of 16 students were recorded in centimetres. Organise the heights into a grouped frequency table.

159 140 145 149 150 134 144 151
155 144 157 141 158 146 132 160

Height, h cm	Frequency
$130 < h \leqslant 140$	3
$140 < h \leqslant 150$	7
$150 < h \leqslant 160$	6
	Total = 16

The line underneath the 'less than' symbols means 'less than or equal to'. Notice that a height of 140 cm is included in the first group.

The test scores of 21 students were recorded. Organise the test scores in an ungrouped frequency table.

6 10 10 9 7 8 7 9 9 5 6 8 7
9 8 10 9 7 9 10 5

Test score	Tally	Frequency
5	||	2
6	||	2
7	||||	4
8	|||	3
9	~~||||~~ |	6
10	||||	4
		Total = 21

The **mean**, **median**, **mode** and **range** can be calculated from a frequency table.

Mean $= \dfrac{\text{sum of all values}}{\text{number of values}}$

$= \dfrac{(5\times2)+(6\times2)+(7\times4)+(8\times3)+(9\times6)+(10\times4)}{21}$

$= 8$

Median $= \dfrac{21+1}{2} = 11\text{th value}$

$= 8$

If the total frequency is odd, find the median by calculating $(n+1) \div 2$, where n is the number of values in the data set and the result is the position of the middle number.

The **mode** is the most common value. This is the test score with the highest frequency.

The **mode** is 9.

Range = highest score − lowest score

$= 10 - 5 = 5$

Two-way frequency tables

A two-way frequency table is used to display data from two different categories.

Remember to include the totals for both categories in a two-way frequency table.

Primary and secondary students were asked if they eat breakfast in the mornings before school.
38 students in primary had breakfast. 12 students in primary did not have breakfast.
22 students in secondary had breakfast. 28 students in secondary did not have breakfast.
Record these results in a two-way frequency table.

	Breakfast	No breakfast	Total
Primary	38	12	50
Secondary	22	28	50
Total	60	40	100

RETRIEVE

6 Frequency tables

Data types and frequency tables

1 Here is a list of the colours of cars in a car park:

green, blue, green, red, red, green, yellow, yellow, blue, red, green, yellow, yellow, yellow, red, green, blue, red, red, yellow, green, yellow, green, blue, yellow, blue, blue, red, green, blue, yellow

Colour	Tally	Frequency
		Total =

a) Is this data quantitative or qualitative?

....

b) Display this data in a frequency table.

2 Here are the speeds of 20 vehicles, to the nearest mph:

45	**65**	**72**	**48**	**74**	**67**	**68**
46	**56**	**53**	**58**	**68**	**72**	**64**
62	**49**	**72**	**55**	**67**	**51**	

Complete the grouped frequency table.

Speed, x mph	Tally	Frequency
$40 < x \leqslant 50$		
$50 < x \leqslant 60$		
$60 < x \leqslant 70$		
$70 < x \leqslant 80$		

3 Here are the temperatures at midday for 7 days (in °C):

23, 24, 24, 23, 24, 25, 21

a) Display this data in an ungrouped frequency table.

b) What is the mean of this data? ....

c) What is the median of this data? ....

d) What is the mode of this data? ....

e) What is the range of this data? ....

Temperature	Tally	Frequency
21°C		
22°C		
23°C		
24°C		
25°C		

Two-way frequency tables

4 Students were asked what they ate for breakfast and lunch.

26 students who had porridge for breakfast had a sandwich for lunch.
14 students who had toast for breakfast had a sandwich for lunch.
10 students who had porridge for breakfast had a hot meal for lunch.
20 students who had toast for breakfast had a hot meal for lunch.

Complete the two-way frequency table.

	Sandwich	Hot meal	Total
Porridge			
Toast			
Total			

ORGANISE

6 Interpreting statistical representations

Interpreting statistical measures and representations

The bar chart shows information about the colour of cars passing a school over a 10-minute period.

a) How many blue cars drove past?

Read off the frequency from the y-axis of the blue car category.

5 blue cars drove past.

b) How many more green cars than silver cars drove past?

Find how many cars of each colour drove past and find the difference between them by **subtracting**.

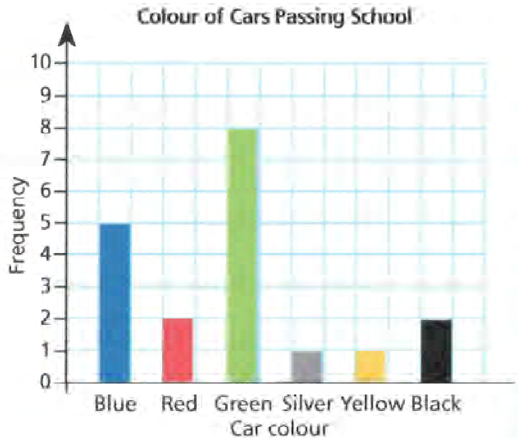
7 more green cars than silver cars drove past.

c) What colour of car drove past the most?

Green cars had the highest frequency, so green cars drove past most often. This is the **mode**.

d) How many cars drove past in total?

19 cars drove past in total. To find the total, add together the frequencies of each colour of car.



Choosing an appropriate statistical measure

When choosing an appropriate statistical measure, consider the advantages and disadvantages of each.

Measure of central tendency	Advantage	Disadvantage
Mean	Takes account of every value	Affected by very large or very small values
Median	Unaffected by very large or very small values	May not be an actual number in the data set
Mode	Only average that can be used for qualitative data	There may be more than one mode or no mode

The salaries of five employees in a company are:

£23000 £25000 £30000 £33000 £120000

Which statistical measure should be used to represent the average?

The **median** should be used because it is unaffected by very large or very small values. £120 000 is a very large value compared to the others.

A shop wants to find the average size of shoe sold to help it to decide which size it needs most stock of. The sizes sold on a particular day are:

3, 4, 4, 5, 6, 7, 8, 8, 9, 9, 9, 9, 9, 10, 10, 11, 12, 14

Which statistical measure should be used for the average?

The **mode** (9) should be used because it shows which shoe size is in greatest demand.

The heights of some Year 8 students, in metres, are:

1.72, 1.54, 1.57, 1.50, 1.55, 1.46, 1.63, 1.61

Which statistical measure should be used for the average height?

The **mean** should be used because it takes account of every value and there are few very large or very small values in the data set.

The range is not a measure of central tendency. It measures the spread of the data set.

The daily temperatures across March last year for two cities are summarised in this table.

City	Mean maximum daily temperature	Range of maximum daily temperature
A	22°C	6°C
B	22°C	13°C

Which city should you choose if you want to enjoy high temperatures? Justify your answer.

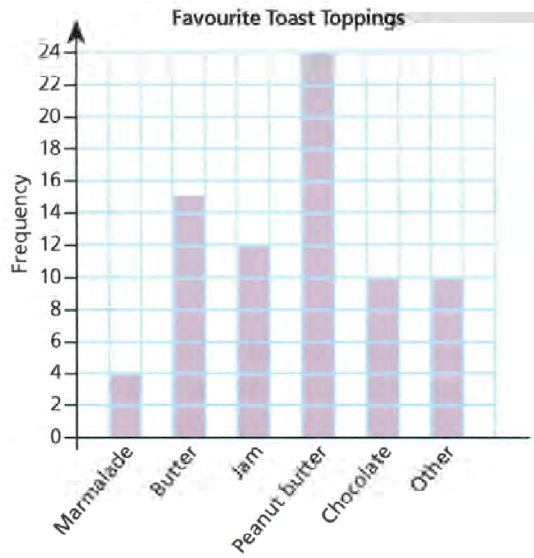
City A should be the city you choose to visit. Both cities have the same mean, but city A has the smaller range. This means that the temperature is more consistently high in city A compared to city B.

RETRIEVE

6 Interpreting statistical representations

Interpreting statistical measures and representations

1 The bar chart shows the preferred toast toppings of a group of students.



a) How many students prefer chocolate on their toast? ....

b) How many students prefer marmalade on their toast? ....

c) How many more students prefer butter on their toast than jam? ....

d) What is the mode of this data? ....

e) How many students took part in the survey? ....

Choosing an appropriate statistical measure

2 A boutique had daily sales of **£326, £540, £385, £450, £2435, £459** and **£493** over the last week.

Is the mean or median a more reliable measure of central tendency? Justify your answer.

....

....

3 The favourite subjects of some students were collected and recorded:

French, PE, Maths, Science, Maths, ICT, Maths, DT, Maths

Which measure of central tendency can best be used to describe this data? Justify your answer.

....

....

4 The daily temperatures across March last year for two cities are summarised in this table.

City	Mean maximum daily temperature	Range of maximum daily temperature
C	12°C	8°C
D	21°C	8°C

Which city should you choose to visit if you want to enjoy high temperatures? Justify your answer.

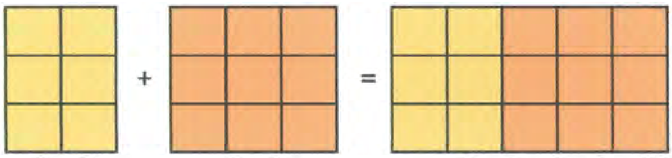
....

....

4 Solving equations with brackets

Expanding brackets using the distributive law

The **distributive law** says that the product of two numbers multiplied together is equal to the product of those numbers split into groups (**partitioned**).



$3 \times 2 + 3 \times 3 = 3 \times 2 + 3 \times 3 = 15$

$3 \times (2 + 3) = 15$

The distributive law can be shown without visual aids by **expanding the bracket**.

$$\begin{aligned} 3(2 + 3) &= (3 \times 2) + (3 \times 3) \\ &= 6 + 9 \\ &= 15 \end{aligned}$$

The same rule applies when there are variables inside or outside the brackets. $5(2x + 3)$ can be expanded using an area model. Draw a large rectangle partitioned into smaller rectangles and find the area of each.

	$2x$	3
5	$10x$	15
	$5 \times 2x$	5×3

$$\begin{aligned} &5(2x + 3) \\ &= (5 \times 2x) + (5 \times 3) \\ &= 10x + 15 \end{aligned}$$

To expand a pair of brackets with more than two terms, you must multiply by each term

To expand $4(2x + 3y - 8)$, multiply 4 by each term in the bracket.

	$2x$	$3y$	-8
4	$8x$	$12y$	-32
	$4 \times 2x$	$4 \times 3y$	4×-8

$$\begin{aligned} &4(2x + 3y - 8) \\ &= 8x + 12y - 32 \end{aligned}$$

Multiply what is outside the brackets by everything inside the brackets.

When expanding brackets, remember that:

- multiplying or dividing two negative numbers gives a positive number
- multiplying or dividing a negative and a positive gives a negative number
- adding a negative number is the same as subtracting
- subtracting a negative number is the same as adding.

$+ \times + = +$

$- \times - = +$

$- \times + = -$

$+ \times - = -$

Expand $3m(2 - 7n)$

Multiply $3m$ by each term inside the bracket and carry the negative sign through to the answer.

$$\begin{aligned} 3m(2 - 7n) &= (3m \times 2) + (3m \times -7n) \\ &= 6m - 21mn \end{aligned}$$

Solving equations with brackets

To solve an equation with one pair of brackets, decide whether it will be easier to expand the brackets first or to divide by the number in front of the brackets.

Solve for k. $3(k - 5) = 6$

$$\begin{aligned} 3 \times k + 3 \times -5 &= 6 \\ 3k - 15 &= 6 \\ +15 \quad &+15 \\ 3k &= 21 \\ \div 3 \quad &\div 3 \\ k &= 7 \end{aligned}$$

$$\begin{aligned} 3(k - 5) &= 6 \\ \div 3 \quad &\div 3 \\ k - 5 &= 2 \\ +5 \quad &+5 \\ k &= 7 \end{aligned}$$

To check, substitute $k = 7$ into $3(k - 5) = 6$

$3(7 - 5) = 3 \times 2 = 6$ ✔

If an equation has more than one pair of brackets, expand all the brackets and simplify before solving.

Solve $3(2j - 4) + 2(j + 3) = 26$

First expand both brackets.

$$\begin{aligned} &3(2j - 4) \\ &= (3 \times 2j) + (3 \times -4) \\ &= 6j - 12 \end{aligned}$$

$$\begin{aligned} &2(j + 3) \\ &= (2 \times j) + (2 \times 3) \\ &= 2j + 6 \end{aligned}$$

$$\begin{aligned} 6j - 12 + 2j + 6 &= 26 \\ 8j - 6 &= 26 \\ +6 \quad &+6 \\ 8j &= 32 \\ \div 8 \quad &\div 8 \\ j &= 4 \end{aligned}$$

RETRIEVE

4 Solving equations with brackets

Expanding brackets using the distributive law

1 Expand each expression and simplify where possible by combining like terms.

a) $3(2k - 4) + 5k$

b) $k(2m - 4)$

c) $4(2x + 3y - 5)$

Solving equations with brackets

2 Solve the following equations.

a) $3(k - 5) = 12$

$k =$ ....

b) $2x + 3(4 - x) = 10$

$x =$ ....

3 Solve the following equations.

a) $2(x + 3) + 4(x - 5) = 10$

$x =$ ....

b) $5(y - 2) = 3(y - 2) + 2$

$y =$ ....

ORGANISE 4

Solving equations with variables on both sides

Using bar models (when all terms are positive)

Bar models are a good way to visualise equations with variables on both sides of the equals symbol when all the terms are positive.

A bar model doesn't work as well when there are negative terms. In that case, it is best to use algebra tiles.

This bar model shows the equation $5x + 1 = 2x + 7$:

<table>
<tr><td colspan="2">x</td><td colspan="2">x</td><td colspan="2">x</td><td colspan="2">x</td><td colspan="2">x</td><td>1</td></tr>
<tr><td colspan="2">x</td><td colspan="2">x</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
</table>

To solve the equation, 'cancel out' the terms that are the same in both bars.

<table>
<tr><td colspan="2">$\cancel{x}$</td><td colspan="2">$\cancel{x}$</td><td colspan="2">x</td><td colspan="2">x</td><td colspan="2">x</td><td>$\cancel{1}$</td></tr>
<tr><td colspan="2">$\cancel{x}$</td><td colspan="2">$\cancel{x}$</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>$\cancel{1}$</td></tr>
</table>

This leaves $3x = 6$, or $1x = 2$, so the answer is $x = 2$.

Using algebra tiles (when terms are positive and/or negative)

To solve equations with variables on both sides using algebra tiles, carry out inverse operations so that all the variable tiles are on one side of the equation and the constants are on the other.

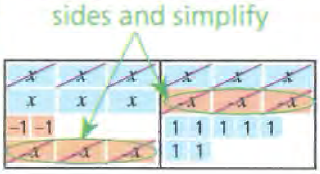
It doesn't matter if the variables are on the left or the right, as long as they are all on the same side.

These algebra tiles show the equation $6x - 2 = 3x + 7$

$6x - 2 = 3x + 7$

Get the variables on one side.

Add 3 $-x$ tiles to both sides and simplify

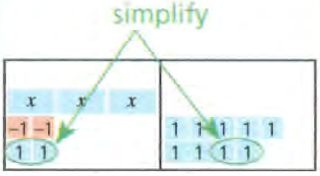


$6x - 2 - 3x = 3x + 7 - 3x$

$3x - 2 = 7$

Isolate the x term then simplify.

Add 2 to both sides and simplify

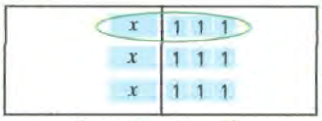


$3x - 2 + 2 = 7 + 2$

$3x = 9$

Divide each side into three equal groups.

1 x tile equals 3 ones tiles



$3x \div 3 = 9 \div 3$

$x = 3$

To check, substitute $x = 3$ into $6x - 2 = 3x + 7$

$6 \times 3 - 2 = 18 - 2 = 16$

$3 \times 3 + 7 = 9 + 7 = 16$ ✔

Solving without visual aids

To solve $2y + 6 = 4y$, use inverse operations to get the y term on one side of the equation. In this case, subtracting $2y$ from both sides is easier because subtracting $4y$ would leave a negative y term, but both methods give the same answer:

$2y + 6 = 4y$

$2y + 6 - 2y = 4y - 2y$ Subtract $2y$ from both sides.

$6 = 2y$ Simplify.

$6 \div 2 = 2y \div 2$ Divide both sides by 2.

$3 = y$

$2y + 6 = 4y$

$2y + 6 - 4y = 4y - 4y$ Subtract $4y$ from both sides.

$-2y + 6 = 0$ Simplify.

$-2y + 6 - 6 = 0 - 6$ Subtract 6 from both sides.

$-2y = -6$ Simplify.

$-2y \div -2 = -6 \div -2$ Divide both sides by -2.

$y = 3$

To check the answer is correct, substitute $y = 3$ into $2y + 6 = 4y$

$2 \times 3 + 6 = 6 + 6 = 12$

$4 \times 3 = 12$ ✔

Similarly, you could subtract 6 from both sides first or divide each term by 2 first and you would arrive at the same answer.

RETRIEVE 4

Solving equations with variables on both sides

Using bar models (when all terms are positive)

1 Draw bar models and solve the following equations.

a) $2x + 8 = 3x + 6$

b) $3b + 5 = 4b + 1$

$x =$ ....

$b =$ ....

Using algebra tiles (when terms are positive and/or negative)

2 Show the equation $4x + 6 = 6x - 4$ using algebra tiles. Then use the tiles or your preferred method to solve the equation. If you do not have algebra tiles to hand, you can cut up bits of paper and label them.

$x =$ ....

Solving without visual aids

3 Solve each of the following equations. Show your working.

a) $2x + 6 = 5x - 15$

$x =$ ....

b) $3x - 4 = 20 - x$

$x =$ ....

ORGANISE

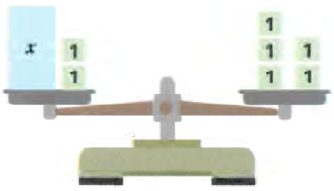
4 Solving one-step equations

Equations and inverse operations

An expression is a collection of **terms** that can be **variables** or **constants** without an equals symbol.

An **equation** consists of one or more **expressions**. Both sides of the equation are equal in value (as shown by the = symbol).

A set of **balance scales** can help to visualise equations. This set of scales represents the equation $x + 2 = 5$.



To solve equations, you need to use **inverse operations** (these 'undo' other operations).

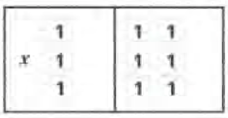
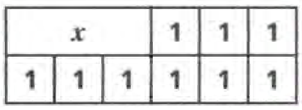
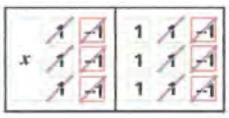
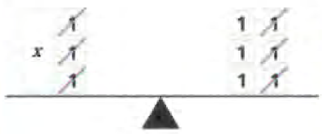
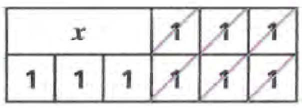
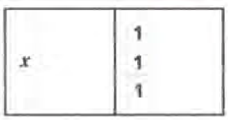
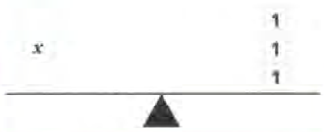
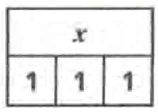
Inverse operations		
Addition +	⇔	Subtraction −
Multiplication ×	⇔	Division ÷
Powers (e.g. x^2, x^3)	⇔	Roots (e.g. $\sqrt{x}$, $\sqrt[3]{x}$)

Using visual aids to solve equations

To '**solve** for x' means to find the value of x. The x term must be **isolated** on one side of the equation. The aim is to get the variables to one side of the equation and the constants to the other.

This equation mat, scales and bar model all show the equation $x + 3 = 6$.

Whatever you do to one side of the equation, you must do to the other to keep it balanced.

Algebraic steps	Equation mat	Scales	Bar model
Lay out the equation. $x + 3 = 6$			
Use inverse operations to isolate x. $x + 3 - 3 = 6 - 3$ Subtract 3 from both sides	Place three −1 tiles on both sides. The +1 and −1 tiles 'cancel out' because they add up to 0. 	Remove 3 ones tiles from both sides. 	'Cancel out' the parts that are the same on both bars. 
Simplify $x = 3$			

Solving without visual aids

To solve an equation without visual aids, use inverses and carry out the same operation on both sides to keep it balanced like a set of scales.

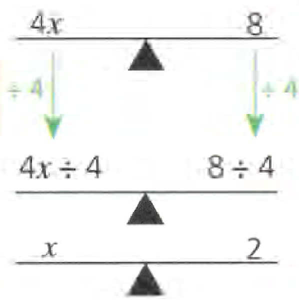
$4x = 8$

The inverse of × 4 is ÷ 4, so divide both sides by 4.

$4x \div 4 = 8 \div 4$

$x = 2$

To check: $4 \times 2 = 8$ ✔



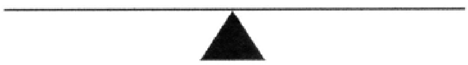
RETRIEVE

4 Solving one-step equations

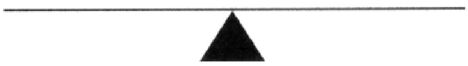
Equations and inverse operations

1 Show each equation on a set of balance scales using algebra tiles.

a) $2x + 4 = 8$



b) $3x + 2 = 11$



Using visual aids to solve equations

2 Show $x + 6 = 8$ using algebra tiles and find the value of x.

$x =$ ....

Solving without visual aids

3 Solve the following equations for x.

a) $x + 9 = 15$

$x =$ ....

b) $x \div 5 = 3$

$x =$ ....

c) $12x = 96$

$x =$ ....

ORGANISE

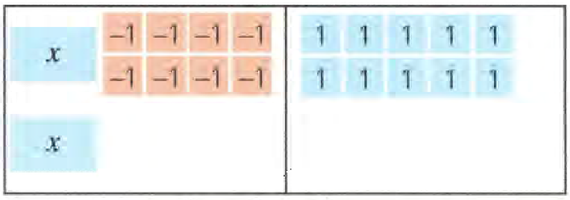
4 Solving two-step equations

Using algebra tiles

Algebra tiles can help to visualise how to solve two-step equations.

Use inverse operations to combine like terms so that all the variables are on one side of the equation and the constants on the other.

Consider $2x - 8 = 10$. There are 2 x tiles and 8 negative one tiles on the left-hand side. There are 10 positive one tiles on the right-hand side.

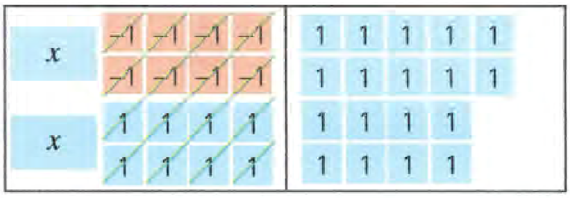


$2x - 8 = 10$

To get x on its own on one side of the equation, use inverse operations and add 8 to both sides.

Simplify by combining the ones tiles.

The 8 positive tiles 'cancel out' the 8 negative ones.

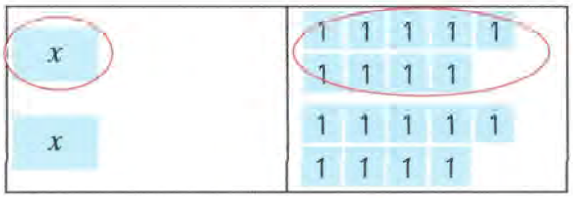


$2x - 8 + 8 = 10 + 8$

$2x = 18$

Now divide each side into two equal groups to find the value of $1x$.

Two x tiles equal 18, so one x tile equals 9.



$2x \div 2 = 18 \div 2$

$x = 9$

To check: $2 \times 9 - 8 = 18 - 8 = 10$ ✔

Solving without visual aids

To solve two-step equations without visual aids, use inverse operations and write down each step. Carry out the same operation on both sides. To solve $2x - 8 = 10$ without algebra tiles, you can first divide by 2 (since all terms are divisible by 2) or you can add 8 to both sides.

$2x - 8 = 10$

$+8 \quad +8$ — Add 8 to both sides.

$2x = 18$

$\div 2 \quad \div 2$ — Divide both sides by 2.

$x = 9$

$2x - 8 = 10$

$\div 2 \quad \div 2$ — Divide both sides by 2.

$x - 4 = 5$

$+4 \quad +4$ — Add 4 to both sides.

$x = 9$

To solve $5x - 9 = 16$, it is not sensible to divide each term before solving because there is not a common factor between 5, 9 and 16.

$5x - 9 = 16$

$+9 \quad +9$ — Add 9 to both sides.

$5x = 25$ — Simplify.

$\div 5 \quad \div 5$ — Divide both sides by 5.

$x = 5$

To check: $5 \times 5 - 9 = 25 - 9 = 16$ ✔

a) Find the value of n in $3n - 5 = 31$

$3n - 5 = 31$

$+5 \quad +5$ — Add 5 to both sides.

$3n = 36$

$\div 3 \quad \div 3$ — Divide both sides by 3.

$n = 12$

To check: $3 \times 12 - 5 = 36 - 5 = 31$ ✔

b) Find the value of a in $\frac{a}{4} - 3 = 2$

Remember that a fraction is another way of writing division, so $\frac{a}{4}$ means $a \div 4$.

$\frac{a}{4} - 3 = 2$

$+3 \quad +3$ — Add 3 to both sides.

$\frac{a}{4} = 5$

$\frac{a}{4} \times 4 = 5 \times 4$ — Multiply both sides by 4.

$a = 20$

To check: $\frac{20}{4} - 3 = (20 \div 4) - 3$

$= 5 - 3 = 2$ ✔

RETRIEVE

4 Solving two-step equations

Using algebra tiles

1 Show the equation $2x - 5 = 9$ using algebra tiles and solve for x.

If you don't have algebra tiles to hand, you can instead cut up bits of paper and label them.

$x =$ ....

Solving without visual aids

2 Solve each of the following equations. Show your working.

a) $6x + 10 = 28$

$x =$ ....

b) $\frac{x}{2} + 3 = 6$

$x =$ ....

c) $5x - 4 = 41$

$x =$ ....

d) $\frac{x}{6} + 8 = 14$

$x =$ ....

Rearranging formulae

The subject of a formula

The subject of a formula is the variable that is isolated, usually to the left of the equals sign.

Identify the subject in each formula.

a) $A = \frac{b \times h}{2}$

A is the subject. The formula shows how the base and height of a triangle are related to the area.

b) $P = 2w + 2l$

P is the subject. The formula shows how the length and width of a parallelogram are related to the perimeter.

It is sometimes useful to rearrange the formula to **change the subject** to a different variable.

Suppose you know the area and height of a triangle and want to find the base. It would be easier if b was the subject of the formula.

How to rearrange formulae

Rearranging formulae is just like solving equations. Use inverse operations to 'undo' the formula so that a different variable is isolated.

You can think of formulae as function machines and work backwards to rearrange the subject. To draw a function machine, think about which variable is the input and which variable is the output. To find the input from the output, work backwards using inverse operations.

Inverse operations		
Addition +	⇔	Subtraction −
Multiplication ×	⇔	Division ÷
Powers (e.g. x^2, x^3)	⇔	Roots (e.g. $\sqrt{x}$, $\sqrt[3]{x}$)

Function machine:

input ⟶ [× 3] ⟶ [+ 4] ⟶ output

Reverse function machine:

input ⟵ [÷ 3] ⟵ [− 4] ⟵ output

Think of rearranging formulae as the same as solving equations. Get the new subject of the formula on its own by using inverse operations.

The formula for the area of a triangle is $A = \frac{bh}{2}$. To rearrange the formula to make the base, b, the subject, think about the operations that are required and apply the inverses.

base ⟶ [× height] ⟶ [÷ 2] ⟶ area

base ⟵ [÷ height] ⟵ [× 2] ⟵ area

To show this algebraically, think about undoing each operation to isolate b.

$A = \frac{bh}{2}$ — Multiply both sides by 2 to 'undo' the fraction.

$A \times 2 = \frac{bh}{2} \times 2$ — Simplify.

$2A = bh$

$\div h \quad \div h$ — Divide both sides by h to solve for b.

$2A \div h = b$

$b = \frac{2A}{h}$ — Rewrite the formula to have the b term (the new subject) on the left-hand side.

Rearrange the formula $V = \pi r^2 h$ to make r the subject.

Using a function machine:

r ⟶ [squared] ⟶ [× π] ⟶ [× h] ⟶ V

Reverse function machine:

r ⟵ [square root] ⟵ [÷ π] ⟵ [÷ h] ⟵ V

Algebraically: $V = \pi r^2 h$ — Divide both sides by πh to isolate the r term.

$\div \pi h \quad \div \pi h$

$\frac{V}{\pi h} = r^2$ — Take the square root of both sides to 'undo' r^2.

$\sqrt{\frac{V}{\pi h}} = \sqrt{r^2}$ — Rewrite the formula to have the r term on the left-hand side.

$r = \sqrt{\frac{V}{\pi h}}$

RETRIEVE

3 Rearranging formulae

The subject of a formula

1 Write down the subject of each formula.

a) $P = sl$ ....

b) $A = \pi r^2$ ....

c) $C = 30h + 45$ ....

How to rearrange formulae

2 You are given the formula $P = sl$, where P is the perimeter of a regular polygon, s is the number of sides and l is the side length.

a) Rearrange the formula to make l the subject.

....

b) Find the length of each side of a 12-sided polygon with perimeter of 108 cm.

....

3 You are given the formula $A = \pi r^2$, where A is the area of a circle and r is the radius.

a) Rearrange the formula to make r the subject.

....

b) Find the radius of a circle with area 225π cm^2.

....

4 You are given the formula $C = 30h + 45$, where C is the charge in £ for a DJ and h is the number of hours worked by the DJ.

a) Rearrange the formula to make h the subject.

....

b) Find the number of hours worked by the DJ if the charge was £135.

....

ORGANISE 7 Rearranging linear equations to $y = mx + c$

Rearranging linear equations

Linear equations can be written in the form $y = mx + c$ where m represents the gradient and c represents the y-intercept.

The gradient describes the steepness of a line, and whether the line slopes upwards or downwards. The y-intercept is the point where the graph crosses the y-axis.

$$y = mx + c$$

m is the gradient of the line

c is the y-intercept

Write down the gradient and the y-intercept of the straight line $y = 3x + 1$

Gradient = 3 $\quad$ y-intercept = 1

Rearrange the equation $2y + 4 = 8x$ into the form $y = mx + c$ and find the gradient and the y-intercept.

$$2y + 4 = 8x$$
$$-4 \qquad -4$$
$$2y = 8x - 4$$
$$\div 2 \qquad \div 2$$
$$y = 4x - 2$$

Gradient = 4 $\quad$ y-intercept = −2

To rearrange linear equations into the form $y = mx + c$, use inverse operations.

Rearrange the equation $2y - 12x = 14$ into the form $y = mx + c$ and find the gradient and the y-intercept.

$$2y - 12x = 14$$
$$+12x \qquad +12x$$
$$2y = 12x + 14$$
$$\div 2 \qquad \div 2$$
$$y = 6x + 7$$

Isolate the variable y so that it is the only term on the left of the equals sign. Use inverse operations to move terms.

Gradient = 6 $\quad$ y-intercept = 7

Rearrange the equation $3y - 6 = 6x + 9$ into the form $y = mx + c$ and find the gradient and the y-intercept.

$$3y - 6 = 6x + 9$$

Add 6 to both sides.

$$+6 \qquad +6$$
$$3y = 6x + 15$$

Divide both sides by 3.

$$\div 3 \quad \div 3 \quad \div 3$$
$$y = 2x + 5$$

Gradient = 2 $\quad$ y-intercept = 5

When dividing the equation by a number, remember to divide all terms by the number.

Equivalent linear equations

To find equivalent linear equations, rearrange the equations so that both are in the form $y = mx + c$.

Which of these equations represents the same straight line as $2x + y = 8$?

$y = 2x + 8$ $\qquad$ $y + 8 = 2x$ $\qquad$ $y = -2x + 8$

Rearrange the equation $2x + y = 8$

$$2x + y = 8$$
$$-2x \qquad -2x$$
$$y = -2x + 8$$

So the answer is $y = -2x + 8$

RETRIEVE 7

Rearranging linear equations to $y = mx + c$

Rearranging linear equations

1 a) Rearrange the equation $2y - 10x = 6$ into the form $y = mx + c$.

....

b) Find the gradient and the y-intercept.

Gradient = ...

y-intercept = ...

2 a) Rearrange the equation $2y + 8 = 6x$ into the form $y = mx + c$.

....

b) Find the gradient and the y-intercept.

Gradient = ...

y-intercept = ...

3 a) Rearrange the equation $2y + 2 = 4x$ into the form $y = mx + c$.

....

b) Find the gradient and the y-intercept.

Gradient = ...

y-intercept = ...

4 a) Rearrange the equation $4y + 3 = 4x - 5$ into the form $y = mx + c$.

....

b) Find the gradient and the y-intercept.

Gradient = ...

y-intercept = ...

5 a) Rearrange the equation $y - 2 = -4x + 7$ into the form $y = mx + c$.

....

b) Find the gradient and the y-intercept.

Gradient = ...

y-intercept = ...

Equivalent linear equations

6 Which of these equations represents the same straight line as $3x + y = 10$? Circle your answer.

$y = -3x + 10$ $\quad$ $y - 3x = 10$ $\quad$ $y = 3x - 10$

7 Which of these equations represents the same straight line as $4x + 2y = 6$? Circle your answer.

$y = -4x + 6$ $\quad$ $y = -2x + 3$ $\quad$ $y = -2x - 6$

ORGANISE

3 Solving multi-step equations 1

Solving one- and two-step equations

Equations are solved using **inverse operations**.

The **one-step equation** $x + 2 = 5$ can be solved as follows:

$$\begin{aligned} x + 2 &= 5 \\ -2 \quad & \quad -2 \\ x &= 3 \end{aligned}$$

Inverse operations		
Addition +	⇔	Subtraction −
Multiplication ×	⇔	Division ÷
Powers (e.g. x^2, x^3)	⇔	Roots (e.g. $\sqrt{x}$, $\sqrt[3]{x}$)

The process is the same for solving **two-step equations**, except you must first isolate the unknown term (usually the variable x) on one side of the equation. It does not matter which side the unknown term is on.

Solve the equation $3x - 8 = 10$

$3x - 8 = 10$
$+8 \quad +8$

Use inverse operations to isolate x. The inverse of − 8 is + 8, so add 8 to both sides.

$3x = 18$
$\div 3 \quad \div 3$
$x = 6$

Now use inverse operations to find the value of x. $3x$ means 3 times x, and the inverse of multiplication is division, so divide both sides by 3.

When solving an equation, carry out the same operation to both sides at each step.

Solving equations with variables on both sides

To solve equations with variables on both sides, isolate the unknown term to one side.

Solve $2x + 8 = 4x - 2$

Subtracting $2x$ from both sides leaves a positive x term, whereas subtracting $4x$ from both sides would leave a negative x term. So it is easier to subtract $2x$.

$2x + 8 = 4x - 2$
$-2x \quad -2x$

Subtract $2x$ from both sides.

$8 = 2x - 2$
$+2 \quad +2$

Use inverse operations to isolate x.

$10 = 2x$
$\div 2 \quad \div 2$

To find the value of x, divide both sides by 2.

$5 = x$
$x = 5$

The answer is usually written with x on the left-hand side.

Solve $7y - 5 = 5y + 3$

$7y - 5 = 5y + 3$
$-5y \quad -5y$

Subtract $5y$ from both sides.

$2y - 5 = 3$
$+5 \quad +5$

Add 5 to both sides.

$2y = 8$
$\div 2 \quad \div 2$

Divide both sides by 2.

$y = 4$

Solving equations with brackets

When an equation has brackets, first expand them, then simplify and proceed to solve the equation as normal.

Solve $2(3x + 8) - 3(x + 4) = 10$

$2(3x + 8) - 3(x + 4) = 10$

First expand $2(3x + 8) = 6x + 16$ and $-3(x + 4) = -3x - 12$

$6x + 16 - 3x - 12 = 10$

Combine like terms.

$3x + 4 = 10$
$-4 \quad -4$

The inverse of + 4 is − 4, so subtract 4 from both sides.

$3x = 6$
$\div 3 \quad \div 3$

$3x$ means $3 \times x$, so divide both sides by 3.

$x = 2$

RETRIEVE

3 Solving multi-step equations 1

Solving one- and two-step equations

1 Solve each equation.

a) $7x + 9 = 51$

$x =$ ....

b) $6x - 9 = 21$

$x =$ ....

Solving equations with variables on both sides

2 Solve each equation.

a) $2x + 8 = 4x - 4$

$x =$ ....

b) $6x - 1 = 15 - 2x$

$x =$ ....

Solving equations with brackets

3 Solve each equation.

a) $3(x - 5) + 2(9 - x) = 10$

$x =$ ....

b) $6x + 7(2x - 3) = 19$

$x =$ ....

ORGANISE

3 Solving multi-step equations 2

Solving equations when the coefficient is negative

When the coefficient of x (or other variable) is negative, manipulate the equation so that the coefficient is positive.

If you have a negative x term in the final step, multiply by −1 to find the value of x.

Solve $10 - 2x = 4$

$$10 - 2x = 4$$
$$+2x \quad +2x$$
$$10 = 4 + 2x$$
$$-4 \quad -4$$
$$6 = 2x$$
$$\div 2 \quad \div 2$$
$$x = 3$$

Manipulate the equation so that $2x$ is isolated on one side and is a positive value.

The answer is usually written with x on the left-hand side.

Solving equations with one fraction

When solving equations with fractions, remember that a fraction is another way of writing division. For example, $\frac{x}{3}$ means $x \div 3$ and $\frac{4}{x}$ means $4 \div x$.

When an equation has just one fraction, simply multiply both sides by the denominator.

When solving equations with fractions and integers, you can either:

- multiply each term by the denominator of the fraction, **or**
- leave the fraction as it is and multiply at the end.

Solve $\frac{4x-2}{5} = 2$

$$\frac{4x-2}{\cancel{5}} \times \cancel{5} = 2 \times 5$$
$$4x - 2 = 10$$
$$+2 \quad +2$$
$$4x = 12$$
$$\div 4 \quad \div 4$$
$$x = 3$$

To 'undo' the fraction, multiply by the denominator.

Solve $4 + \frac{x}{3} = 8$

Method 1: Multiply each term by the denominator first.

$$(4 \times 3) + (\tfrac{x}{3} \times 3) = (8 \times 3)$$
$$12 + x = 24$$
$$-12 \quad -12$$
$$x = 12$$

Method 2: Isolate the x term first.

$$4 + \frac{x}{3} = 8$$
$$-4 \quad -4$$
$$\frac{x}{3} = 4$$
$$\times 3 \quad \times 3$$
$$x = 12$$

Solving equations with fractions on both sides

When an equation has a fraction on both sides, first find the lowest common multiple (LCM) of the two denominators, then multiply by the LCM to 'undo' them.

Solve $\frac{3x+1}{2} = \frac{5x+5}{4}$

$$\frac{3x+1}{2} \times 4 = \frac{5x+5}{4} \times 4$$
$$\frac{(3x+1) \times {}^{2}\cancel{4}}{\cancel{2}} = \frac{(5x+5) \times \cancel{4}}{\cancel{4}}$$
$$2(3x+1) = 5x + 5$$
$$6x + 2 = 5x + 5$$
$$-2 \quad -2$$
$$6x = 5x + 3$$
$$-5x \quad -5x$$
$$x = 3$$

The LCM of 2 and 4 is 4. Multiply both sides by 4.

Expand the brackets.

RETRIEVE

3 Solving multi-step equations 2

Solving equations when the coefficient is negative

1 A student is trying to solve the equation $4 - 3x = -2$

The student has made a mistake. Rewrite the solution correctly.

$$\begin{aligned} 4 - 3x &= -2 \\ -4 \quad & \quad -4 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$

Solving equations with one fraction

2 Solve each equation.

a) $\frac{3k-2}{4} = 4$

$k =$ ....

b) $2 + \frac{x+1}{3} = 5$

$x =$ ....

Solving equations with fractions on both sides

3 Solve the equation.

$\frac{3-2m}{2} = \frac{m+2}{6}$

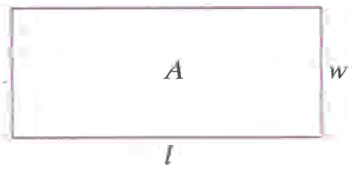
$m =$ ....

ORGANISE

3 Using and writing formulae

Writing formulae

A **formula** is a rule that shows how two or more **variables** are linked. The variable that is being worked out is called the **subject**.



$A = l \times w$ is a formula. It shows how the variables of length, width and area of a rectangle are related. A is the subject.

For example, a plumber charges a £100 call-out fee plus an additional £25 for every hour worked:

- In words, to calculate the cost, multiply £25 by the number of hours worked and add that to the call-out fee of £100.
- Writing the formula algebraically, state the variables. Let C = the total cost the plumber charges and h = the number of hours worked. Then the formula is $C = 100 + 25h$

Cost (the subject) Number of hours (the variable)
Call-out fee Price per hour

A formula shows how two or more variables are related. When writing a formula algebraically, make sure you state what each variable represents.

Using formulae

To use a formula, substitute in the given values of the known variables and find the unknown variable.

A taxi company charges a £5 flat rate plus £1.25 per mile.

a) Write the formula in words.

Multiply the cost per mile, £1.25, by the number of miles travelled and add the product to the flat rate of £5.

b) Write the formula algebraically.

$C = 5 + 1.25m$, where C = the total cost and m = the number of miles travelled

c) Find the cost of taking a taxi for 8 miles.

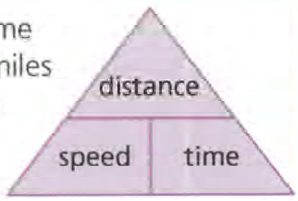
$C = 5 + 1.25m$

$C = 5 + (1.25 \times 8)$

$C = £15$

To find the cost for travelling 8 miles, substitute $m = 8$.

Speed is the distance travelled per unit of time (e.g. 50 miles per hour means travelling 50 miles in one hour). This relationship can be shown using a formula triangle.



To use the triangle, cover the subject and multiply or divide the other variables as appropriate.

The triangle shows three formulae:
Speed = distance ÷ time
Time = distance ÷ speed
Distance = speed × time

A plane travels at an average speed of 900 km/h for 8.5 hours. How far has it travelled?

Use the formula triangle, $D = S \times T$, where D = Distance, S = Speed and T = Time.

$D = S \times T$

$D = 900 \times 8.5$

$D = 7650\text{km}$

Substitute $T = 8.5\text{h}$ and $S = 900\text{km/h}$.

The formula $C = \frac{5(F-32)}{9}$ can be used to convert between degrees Celsius (°C) and degrees Fahrenheit (°F).

Find the temperature in Celsius when it is 85°F, giving the answer to 1 decimal place.

Substitute $F = 85$ into the formula $C = \frac{5(F-32)}{9}$

$C = \frac{5(85-32)}{9}$ First work out the brackets, 85 − 32 = 53

$= \frac{5 \times 53}{9}$ A number written outside brackets means to multiply. 5 × 53 = 265

$= \frac{265}{9}$ A fraction is another way of writing division, so divide. 265 ÷ 9 = $29.\dot{4}$

$= 29.4$°C (to 1 d.p.)

RETRIEVE

3 Using and writing formulae

Writing formulae

1 Write a formula in words and algebraically for each situation.

a) Finding the perimeter of a regular polygon.

In words: ....

Algebraically: ....

b)

In words: ....

Algebraically: ....

Using formulae

2 Use the formulae from question 1 to find the following.

a) The perimeter of a regular decagon (10 sides) with side lengths of 1.3 cm

.... cm

b) The cost of a party with 25 guests

£ ....

3 A train travels 630 km at a speed of 280 km/h.

How many hours does the journey take?

....