

Substitution

To **substitute** in an expression means to replace a variable with a number to evaluate the expression.

Use brackets when substituting to make sure you obey the order of operations.

a) Evaluate a^3 when $a = 4$

$$\begin{aligned} a^3 &= a \times a \times a \\ &= 4 \times 4 \times 4 = 64 \\ \text{So } a^3 &= 64 \end{aligned}$$

b) Evaluate $2x + 4$ when $x = 6$

$$\begin{aligned} 2x + 4 &= (2 \times x) + 4 \\ &= (2 \times 6) + 4 = 12 + 4 = 16 \\ \text{So } 2x + 4 &= 16 \end{aligned}$$

Evaluate $5x + 3y$ when $x = 2$ and $y = 7$

$$\begin{aligned} 5x + 3y &= (5 \times x) + (3 \times y) \\ &= (5 \times 2) + (3 \times 7) \\ &= 10 + 21 = 31 \\ \text{So } 5x + 3y &= 31 \end{aligned}$$

Which set of variables satisfy $a + 13 = b$?

$$a = 8, b = 22$$

or

$$a = 7, b = 20$$

$$\begin{aligned} 8 + 13 &= b \\ 8 + 13 &= 22 \quad \times \end{aligned}$$

$$\begin{aligned} 7 + 13 &= b \\ 7 + 13 &= 20 \quad \checkmark \end{aligned}$$

So $a = 7$ and $b = 20$

Forming algebraic expressions and equations

Algebraic expressions and equations can be formed from statements and shapes.

Use algebra to show the relationships.

a) Two different numbers add to 10.

$$x + y = 10$$

These are two different numbers so they are represented by two different variables.

b) Two numbers are added together to make a third number.

$$a + b = c$$

Two different numbers form a third number, so use three different variables.

c) Two of the same number are added together to make another number.

$$n + n = m \text{ or } n + n = 2n$$

Numbers that are the same are represented using the same variable. The result is a different number so is represented by a different variable.

Form an algebraic equation to represent the following statement:

I am thinking of a number. I add 4 to my number and multiply by 5. I get 30.

The variable y can represent the unknown number.

"I add 4 to my number": $y + 4$

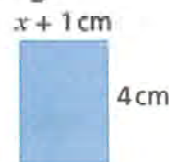
"and multiply by 5": $5(y + 4)$

Remember to use brackets when multiplying an expression by a number.

"I get 30": $5(y + 4) = 30$

So $5(y + 4) = 30$

Form an expression for the area of a rectangle with these side lengths.



$$\begin{aligned} \text{Area of a rectangle} &= \text{length} \times \text{width} \\ &= 4 \times (x + 1) \\ &= 4(x + 1) \text{ cm}^2 \end{aligned}$$

Interpreting algebraic expressions

Break down each part of the expression to interpret it algebraically.

$x + 5$ means 'increase x by 5' or '5 more than x '.

$7 + y$ means 'increase 7 by y ' or ' y more than 7'.

$q - 3$ means 'decrease q by 3' or '3 less than q '.

$3 - q$ means 'decrease 3 by q ' or ' q less than 3'.

$p \times 5 = 5p$ means '5 times as much as p '.

$5 \times p = 5p$ means ' p times as much as 5'.

$s \div 3$ means 'divide s by 3' or 'one third of s '.

$3 \div s$ means 'divide 3 by s ' or 'an s^{th} of 3'.

Algebraic expressions

Substitution

1 Evaluate:

- a) $7s$ when $s = 2$ b) t^2 when $t = 6$
- c) u^3 when $u = 5$ d) $6v - 2$ when $v = 8$
- e) $8v - 3y$ when $v = 2$ and $y = 4$

2 Determine which set of variables satisfy the equation $y - 16 = z$

$$y = 9, z = -7$$

or

$$y = -20, z = -4$$

Forming algebraic expressions and equations

3 Use algebra to represent each of these relationships.

- a) Two different numbers add to 20
- b) The difference between two numbers is 3
- c) Three identical numbers are multiplied together to make another number

4 Using n as the variable, form an algebraic expression from each statement.

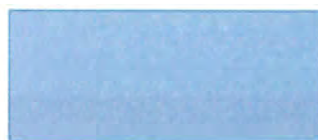
- a) Three times Leo's age
- b) Nine years older than Fatima
- c) Six years younger than Amira

5 Use algebra to represent each statement.

- a) *I am thinking of a number.*
I multiply my number by 4 and subtract 1.
- b) *I am thinking of a number.*
I add 6 to my number, and divide by 4. I get 20.

6 Form an expression for the area of a rectangle with these side lengths.

$$x + 4 \text{ cm}$$



$$3 \text{ cm}$$

Interpreting algebraic expressions

7 Draw lines to join each expression to the correct interpretation.

$$3b$$

$$b \div 2$$

$$2 - b$$

$$b + 3$$

3 more than b Decrease 2 by b 3 times as much as b Half of b

Algebraic vocabulary

Algebra is used to represent missing information.

An algebraic x is written using two back-to-back 'c's.

Vocabulary	Examples
A letter can be used to represent any unknown number. This is called a variable .	x or y
A number that multiplies a variable is called a coefficient .	$3x$ or $2y$
A constant is a number that does not change in value.	5 or -8
A term is a single variable or a single constant or variables and constants that have been multiplied together.	a $3b$ 12
An expression is two or more terms joined by an operator (e.g. $+$, $-$, $\times$). In an expression that is a product, the terms that are being multiplied are called factors .	$x - 4 + 3y$ In the expression $7 \times a$, 7 and a are factors.
An equation shows that two expressions are equal (indicated by the $=$ sign).	$7y = 21$
A formula is a variable that is equal to an expression which uses a different variable.	$s = \frac{d}{t}$

In the expression $2x + 11 - 8y$, notice that:

- the **variables** are x and y
- the **coefficients** are 2 and -8
- the **constant** is 11
- the **terms** are $2x$, 11 and $-8y$

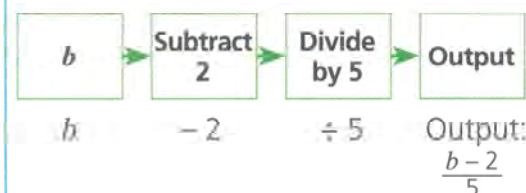
When a variable and a number are multiplied together, the number is always written first. Both $z \times 5$ and $5 \times z$ are written as $5z$.
A product is the result of a multiplication.

Algebraic notation and function machines

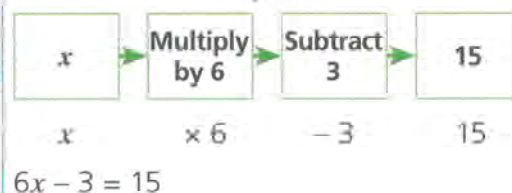
Rules of algebraic notation	Examples
When multiplying in algebra, the $\times$ sign is not written.	$x \times y$ is written as xy $6 \times (t + 1)$ is written as $6(t + 1)$
When dividing in algebra, the division is usually expressed as a fraction.	$x \div y$ is written as $\frac{x}{y}$ $a \div 3$ is written as $\frac{a}{3}$
When multiplying a variable by 1 , the number 1 is not written as the coefficient.	$1 \times w$ is written as w
When multiplying a variable by 0 , the number 0 is not written as the coefficient.	$0 \times w$ is written as 0 , not $0w$
When multiplying a variable by itself, powers are used.	$y \times y$ is written as y^2 $y \times y \times y$ is written as y^3

A **function machine** applies operations to a variable, called the **input**. The result is called the **output**.

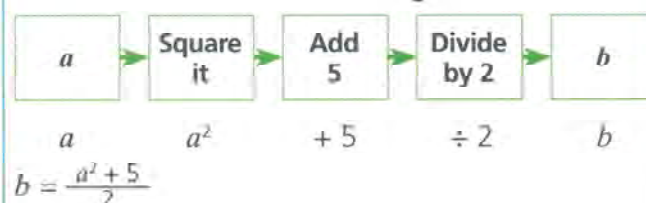
Write the output of the function machine as an **expression**.



Write the instructions in the function machine as an **equation**.



Write the instructions in the function machine as a **formula** linking b and a .



Squaring a variable means to multiply the variable by itself, e.g. " a squared" = a^2

Cubing a variable means to multiply the variable by itself and by itself again, e.g. " a cubed" = a^3

Algebraic vocabulary and notation

Algebraic vocabulary

1 Identify the variables, coefficients, constants and terms in each expression.

- a) $3x + y - 12$ Variables: _____ Coefficients: _____
 Constants: _____ Terms: _____
- b) $-6f + g + 5$ Variables: _____ Coefficients: _____
 Constants: _____ Terms: _____
- c) $10 - de + 4f$ Variables: _____ Coefficients: _____
 Constants: _____ Terms: _____

2 Identify the factors in each expression.

- a) $b \times 12$ _____
- b) $5 \times \frac{1}{n}$ _____

3 Identify whether each of these is an **expression**, an **equation** or a **formula**.

- a) $5a + 4 = 20$ _____
- b) $u = x + 2xy - 4$ _____
- c) $7e + fg - 18$ _____

Algebraic notation and function machines

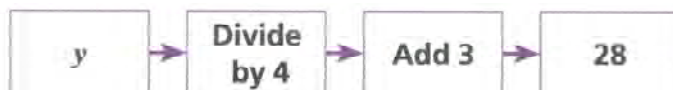
4 Write the expressions algebraically.

- a) $f \times 3$ _____ b) $w \times x \times y$ _____ c) $8 \times (c - d)$ _____
- d) $m \div n$ _____ e) $7 \div a$ _____ f) $b \div 9$ _____
- g) $1 \times d$ _____ h) $h \times 0$ _____ i) $n \times n$ _____

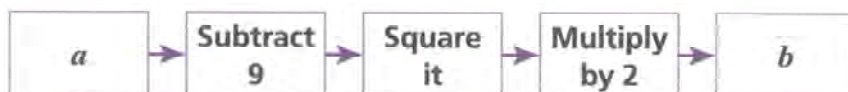
5 a) Write the instructions in the function machine as an **expression**.



b) Write the instructions in the function machine as an **equation**.



c) Write the instructions in the function machine as a **formula** linking b and a .



Ratio notation

Ratios are used to compare quantities.

: means 'to' in ratio.

B B B B B Y Y Y

The ratio of black counters to yellow counters is 5 : 3

This means that for every 5 black counters, there are 3 yellow counters.

The ratio of yellow counters to black counters is 3 : 5

This means that for every 3 yellow counters, there are 5 black counters.

The part that is written first is the quantity that should be written first in the ratio.



There are 4 butterflies and 2 snails.

The ratio of butterflies to snails is 4 : 2

The ratio of snails to butterflies is 2 : 4

Ratios can have more than two parts.

G G G G B B B R R

The ratio of green counters to blue counters to red counters is 4 : 3 : 2

Writing ratios

In one month, there were 12 sunny days and 18 rainy days.

The ratio of sunny days to rainy days was 12 : 18

The ratio of rainy days to sunny days was 18 : 12

When ratios have common factors, they can be simplified by dividing both numbers by the highest common factor. For example, 12 : 18 could be simplified to 2 : 3 by dividing both sides by 6. Page 58 covers this in more detail.

Ratios and fractions

Ratios can be expressed as fractions and fractions can be expressed as ratios.

Ratios compare quantities

Quantity ← $\square : \square$ → Quantity

Troy and Anji share some money in the ratio 3 : 4. What fraction of the money does Troy get?

Number of parts that Troy gets is 3

Total number of parts is $3 + 4 = 7$

Fraction of money that Troy gets is $\frac{3}{7}$

Fractions compare part of a quantity to the whole

$\frac{\square}{\square}$ ← Quantity
← Total amount

To find a missing part in a ratio from a fraction, subtract the quantity given in the numerator of the fraction from the total amount in the denominator.

The marbles in a bag are blue, red, and yellow. They are in the ratio 1 : 2 : 5.
What fraction of the marbles are red?

Number of parts that are red is 2

Total number of parts is $1 + 2 + 5 = 8$

Fraction of marbles that are red is $\frac{2}{8} = \frac{1}{4}$

A shopping bag contains some fruits and vegetables. The fraction of vegetables in the bag is $\frac{4}{9}$. Find the ratio of fruits to vegetables.

Number of parts that are vegetables is 4

Number of parts that are fruits is $9 - 4 = 5$

Ratio of fruits to vegetables is 5 : 4

Ratio notation

- 1 Write the number of yellow counters to blue counters as a ratio.



- 2 Write the number of red cars to blue cars as a ratio.



- 3 Write the number of rectangles to triangles as a ratio.



- 4 Write the number of red cubes to yellow cubes to grey cubes as a ratio.



Writing ratios

- 5 There are 30 chairs and 15 tables in a classroom.
Write the ratio of chairs to tables.
- 6 A bag contains 8 orange sweets and 15 red sweets.
a) Write the ratio of orange sweets to red sweets.
b) Write the ratio of red sweets to orange sweets.
- 7 A bus is carrying 22 seated passengers and 16 standing passengers.
a) Write the ratio of seated passengers to standing passengers.
b) Write the ratio of standing passengers to seated passengers.
- 8 A team plays 24 matches. 10 are wins, 6 are draws and 8 are defeats.
Write the ratio of wins to draws to defeats.

Ratios and fractions

- 9 A bag has black and yellow counters in the ratio 7 : 4.
What fraction of the counters are yellow?
- 10 Pancake mix is made by mixing milk, flour, and eggs in the ratio 3 : 3 : 1.
What fraction of the pancake mix is flour?
- 11 Viv bought some jumpers and shirts. $\frac{1}{9}$ of his shopping was jumpers.
Write the ratio of jumpers to shirts.

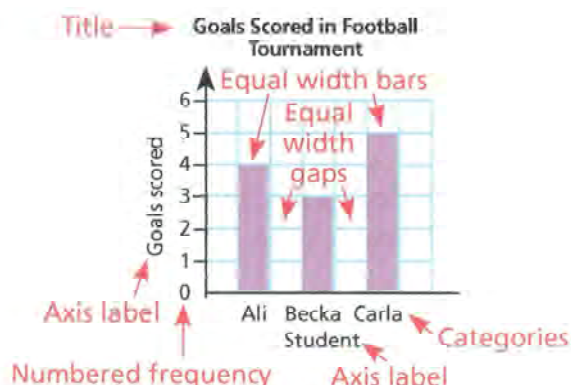
Bar charts and pictograms

Bar charts

A bar chart is used to display data using rectangular bars of different heights or lengths.

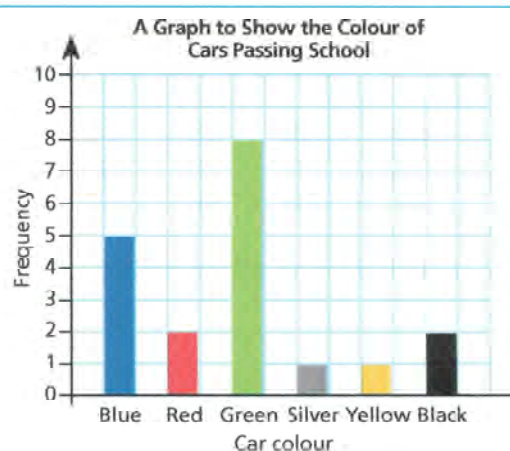
A bar chart must include:

- a title
- labels on both axes
- numbered frequency
- bars of equal width
- categories
- gaps of equal width between the bars.



This table shows the colours of cars that passed a school. Draw a bar chart for this information.

Colour	Frequency
Blue	5
Red	2
Green	8
Silver	1
Yellow	1
Black	2

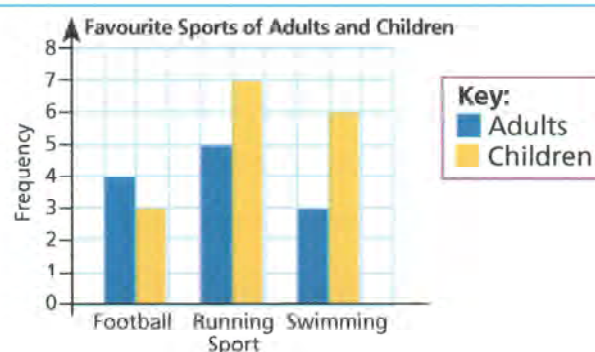


Dual bar charts show a comparison between two or more sets of data.

Dual bar charts must include a key to identify the different categories.

This two-way frequency table shows the favourite sports of some adults and children. Draw a dual bar chart for this information.

	Football	Running	Swimming
Adults	4	5	3
Children	3	7	6



Pictograms

A pictogram can also represent data. Unlike a bar chart, a pictogram uses rows or columns of pictures to represent frequency.

Half a picture represents half of the frequency in the key.

The table below shows the favourite flavour of crisps of some students. Draw a pictogram for this information.

Crisp flavour	Frequency
Ready salted	28
Cheese and onion	16
Salt and vinegar	24
Chicken	10

A Pictogram of Favourite Crisp Flavours

Crisp flavour	Number of students
Ready salted	☺☺☺☺☺☺☺☺
Cheese and onion	☺☺☺☺
Salt and vinegar	☺☺☺☺☺☺☺
Chicken	☺☺☺☺

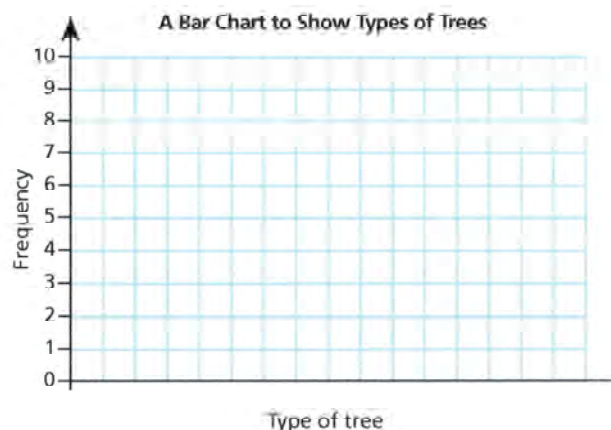
Key: ☺ represents 4 students

Bar charts and pictograms

Bar charts

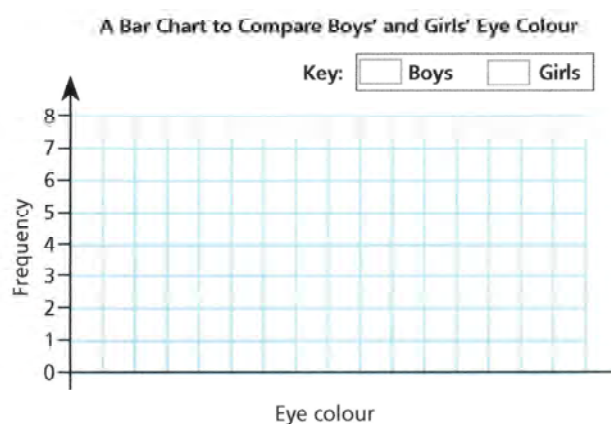
- 1 The types of trees in a particular area were recorded in the frequency table.
- Draw a bar chart to represent this information.

Tree	Frequency
Oak	2
Birch	5
Evergreen	8
Pine	10
Cedar	7



- 2 The eye colours of some boys and girls were recorded in the frequency table.
- Draw a dual bar chart to represent this information.

Eye colour	Boys	Girls
Blue	8	7
Green	4	4
Brown	2	4



Pictograms

- 3 The table below shows the favourite sports of some Year 8 students.
- Complete the pictogram to represent this information.

Sport	Frequency
Badminton	20
Netball	15
Basketball	25
Table tennis	5

Favourite Sports of Year 8 Students

Badminton	
Netball	
Basketball	
Table tennis	
Key: = 5	

- 4 The table below shows the number of cupcakes sold by a shop over 6 days.
- Complete the pictogram to represent this information.

Day	Frequency
Monday	30
Tuesday	15
Wednesday	24
Thursday	21
Friday	42
Saturday	57

Number of Cupcakes Sold

Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Key: = 6 cupcakes	



Frequency tables

Data types and frequency tables

Data is a set of information that can be analysed. Data can be collected by using tables, which makes it easier to read and understand.

Qualitative data is data that describes something (e.g. colour of hair; colour of eyes).

Quantitative data is data that is numerical (e.g. height; shoe size).

There are two types of quantitative data:

- **Discrete** data can only take certain values (e.g. the number of people in a room; the number of pages in a book).
- **Continuous** data can take any value (e.g. the time taken to run a race; the mass of an object).

A **grouped** frequency table is used to organise and simplify a large set of data into smaller groups. A frequency table containing **ungrouped** data has data that has not been put into categories or simplified.

The heights of 16 students were recorded in centimetres. Organise the heights into a grouped frequency table.

159 140 145 149 150 134 144 151
155 144 157 141 158 146 132 160

Height, h cm	Frequency
$130 < h \leq 140$	3
$140 < h \leq 150$	7
$150 < h \leq 160$	6
Total = 16	

The line underneath the 'less than' symbols means 'less than or equal to'. Notice that a height of 140 cm is included in the first group.

The test scores of 21 students were recorded. Organise the test scores in an ungrouped frequency table.

6 10 10 9 7 8 7 9 9 5 6 8 7
9 8 10 9 7 9 10 5

Test score	Tally	Frequency
5		2
6		2
7		4
8		3
9		6
10		4
		Total = 21

The **mean**, **median**, **mode** and **range** can be calculated from a frequency table.

$$\begin{aligned} \text{Mean} &= \frac{\text{sum of all values}}{\text{number of values}} \\ &= \frac{(5 \times 2) + (6 \times 2) + (7 \times 4) + (8 \times 3) + (9 \times 6) + (10 \times 4)}{21} \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{Median} &= \frac{21 + 1}{2} = 11\text{th value} \\ &= 8 \end{aligned}$$

The **mode** is the most common value. This is the test score with the highest frequency.

The **mode** is 9.

$$\begin{aligned} \text{Range} &= \text{highest score} - \text{lowest score} \\ &= 10 - 5 = 5 \end{aligned}$$

If the total frequency is odd, find the median by calculating $(n + 1) \div 2$, where n is the number of values in the data set and the result is the position of the middle number.

Two-way frequency tables

A two-way frequency table is used to display data from two different categories.

Remember to include the totals for both categories in a two-way frequency table.

Primary and secondary students were asked if they eat breakfast in the mornings before school. 38 students in primary had breakfast. 12 students in primary did not have breakfast. 22 students in secondary had breakfast. 28 students in secondary did not have breakfast. Record these results in a two-way frequency table.

	Breakfast	No breakfast	Total
Primary	38	12	50
Secondary	22	28	50
Total	60	40	100

Frequency tables

Data types and frequency tables

- 1 Here is a list of the colours of cars in a car park:

green, blue, green, red, red, green, yellow,
yellow, blue, red, green, yellow, yellow,
yellow, red, green, blue, red, red, yellow,
green, yellow, green, blue, yellow, blue,
blue, red, green, blue, yellow

- a) Is this data quantitative or qualitative?

Colour	Tally	Frequency
Total =		

- b) Display this data in a frequency table.

- 2 Here are the speeds of 20 vehicles, to the nearest mph:

45 65 72 48 74 67 68
46 56 53 58 68 72 64
62 49 72 55 67 51

Complete the grouped frequency table:

Speed, x mph	Tally	Frequency
$40 < x \leq 50$		
$50 < x \leq 60$		
$60 < x \leq 70$		
$70 < x \leq 80$		
Total =		

- 3 Here are the temperatures at midday for 7 days (in $^{\circ}\text{C}$):

23, 24, 24, 23, 24, 25, 21

- a) Display this data in an ungrouped frequency table.
b) What is the mean of this data? _____
c) What is the median of this data? _____
d) What is the mode of this data? _____
e) What is the range of this data? _____

Temperature	Tally	Frequency
21°C		
22°C		
23°C		
24°C		
25°C		
Total =		

Two-way frequency tables

- 4 Students were asked what they ate for breakfast and lunch.

26 students who had porridge for breakfast had a sandwich for lunch.

14 students who had toast for breakfast had a sandwich for lunch.

10 students who had porridge for breakfast had a hot meal for lunch.

20 students who had toast for breakfast had a hot meal for lunch.

Complete the two-way frequency table:

	Sandwich	Hot meal	Total
Porridge			
Toast			
Total			

Pie charts and scatter graphs

Pie charts

Pie charts use a circle to display data. Pie charts show the **proportion** or **fraction** of each category within the data compared to the whole circle.

To find the size of each angle, multiply the fraction by 360 because there are 360° in a circle. Always label the sections of a pie chart.

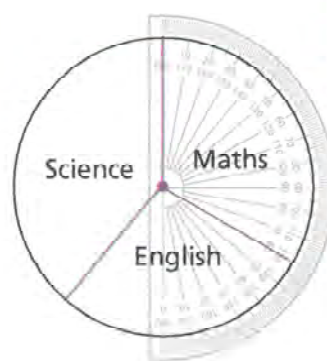
The frequency table (right) shows the favourite subject of 36 students.

Draw a pie chart to show this data.

To find the size of each angle, express each subject's frequency as a fraction of the total number of students asked, and multiply this by 360°.

Subject	Frequency	Fraction	Angle
Maths	12	$\frac{12}{36}$	120°
English	10	$\frac{10}{36}$	100°
Science	14	$\frac{14}{36}$	140°

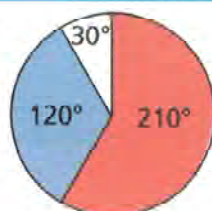
Subject	Frequency
Maths	12
English	10
Science	14



To draw a pie chart, use a **protractor** to mark off the angles that are needed.

You can find the frequency of a category if given the total frequency and the size of an angle in a pie chart.

The pie chart shows the proportion of different fish in a tank. There are 24 fish in total.



Key:
 Blue fish
 White fish
 Red fish

Work out how many blue fish are in the tank.

$$\frac{120}{360} \times 24 = 8$$

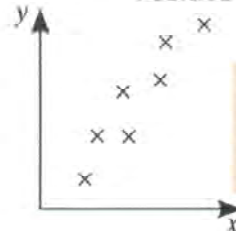
There are 8 blue fish in the tank.

Scatter graphs

Scatter graphs show the relationship between two sets of **numerical** data.

The relationship is described using **correlation**. There are three types of correlation:

Positive



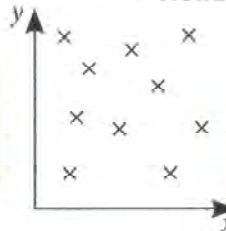
As one increases the other increases

Negative



As one increases the other decreases

None



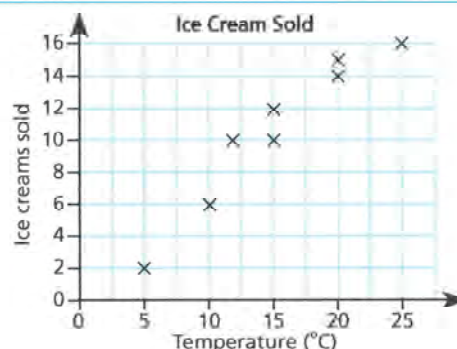
No pattern or relationship

Correlation is strong if the points on a scatter graph closely follow a straight line. Correlation is weak if the points are more loosely spread from a straight line.

A shopkeeper records the temperature for 8 days and the number of ice creams sold.

Temperature (°C)	Ice creams sold
20	14
15	10
25	16
10	6
20	15
5	2
15	12
12	10

a) Plot the data on a scatter graph.



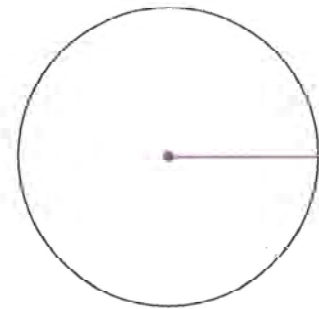
b) What type of correlation does the graph show?
 A positive correlation. The higher the temperature, the more ice creams that are sold.

Pie charts and scatter graphs

Pie charts

- 1 The frequency table shows the favourite colour of 36 students.

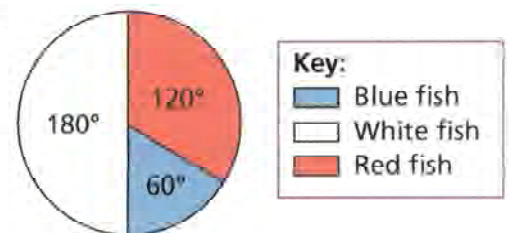
Colour	Frequency	Fraction	Angle
White	9		
Green	20		
Red	7		



- a) Complete the table.
b) Draw a pie chart to represent this information.

- 2 The pie chart shows the proportion of different fish in a tank. There are 48 fish in total.

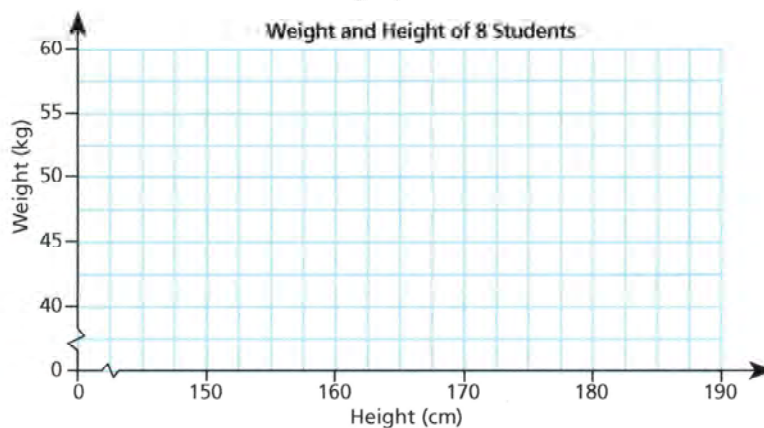
- a) Work out the number of blue fish in the tank. _____
b) Work out the number of white fish in the tank. _____
c) Work out the number of red fish in the tank. _____



Scatter graphs

- 3 The height and weight of 8 students are shown in the table.

- a) Plot the data on a scatter graph.

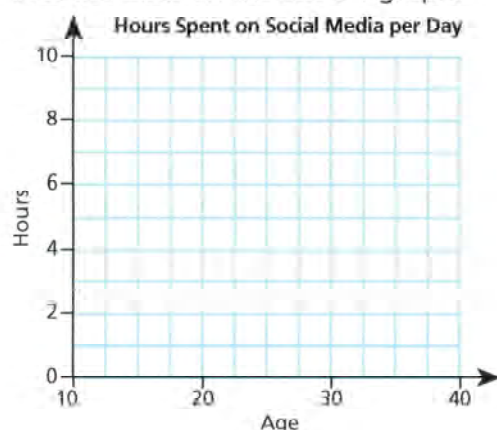


Height (cm)	Weight (kg)
150	45
156	46
156	51
181	57
170	51
172	58
165	52
180	50

- b) Describe the correlation between height and weight.

- 4 The number of hours spent on social media by people of different ages is shown in the table.

- a) Plot the data on the scatter graph.



Age	Hours spent on social media per day
10	10
15	8
21	6
27	4
32	2

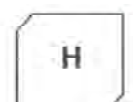
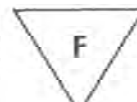
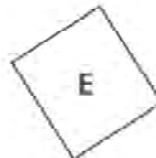
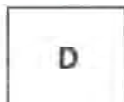
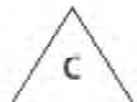
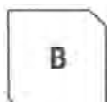
- b) Describe the correlation between age and the number of hours spent on social media in a day.

Congruent shapes

Congruent shapes are **identical**. They are exactly the same size and exactly the same shape.

If a shape is reflected, rotated or translated, the image is congruent to the original shape. The corresponding sides and the corresponding angles stay the same.

Which pairs of shapes are congruent?



The following pairs of shapes are congruent:

A and G

B and H

C and F

D and E

In each pair, the shapes are identical in size and identical in shape.

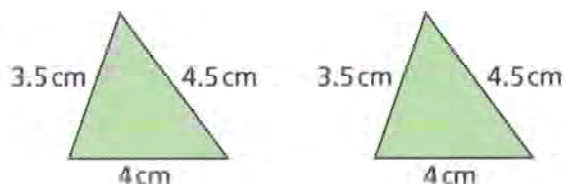
Congruent triangles

Two triangles are congruent if one or more of these four criteria are true.

Side, Side, Side (SSS)	Side, Angle, Side (SAS)	Angle, Side, Angle (ASA)	Right angle, Hypotenuse, Side (RHS)
The three sides are equal.	Two sides are equal and the angle between them is equal.	Two angles are the same size and a corresponding side is the same length.	A right angle, the hypotenuse and a corresponding side are equal.

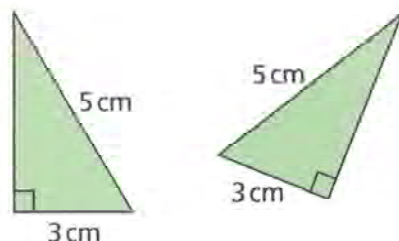
Decide if the triangles in each pair are congruent, giving a reason in each case.

a)



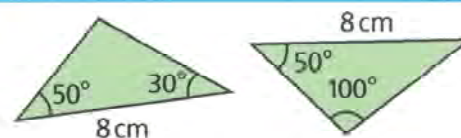
Yes, the triangles are congruent. All three sides are equal (SSS).

b)



Yes, the triangles are congruent. They each have a right angle, they have equal hypotenuses and one other corresponding side is also equal (RHS).

c)



Yes, the triangles are congruent.

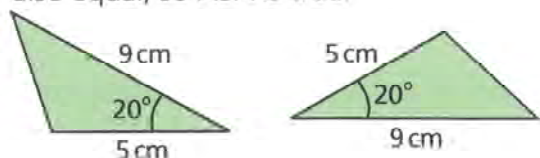
Angles in a triangle sum to 180° .

In the first triangle, $50^\circ + 30^\circ = 80^\circ$ and $180^\circ - 80^\circ = 100^\circ$ so the unmarked angle is 100° .

In the second triangle, $100^\circ + 50^\circ = 150^\circ$ and $180^\circ - 150^\circ = 30^\circ$ so the unmarked angle is 30° .

Therefore, the corresponding angles are equal. A corresponding side (the side opposite the 100° angle in each triangle) is also equal, so ASA is true.

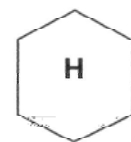
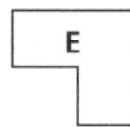
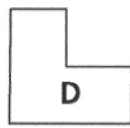
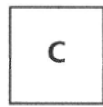
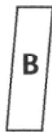
d)



Yes, the triangles are congruent. They have two equal sides and the angle between them is equal (SAS).

Congruent shapes

- 1 Find four pairs of congruent shapes.



_____ and _____

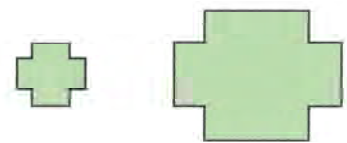
_____ and _____

_____ and _____

_____ and _____

- 2 Are these two shapes congruent?

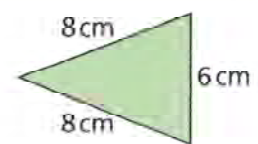
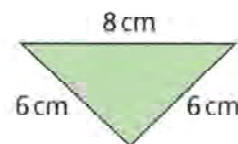
Give a reason to support your answer.



Congruent triangles

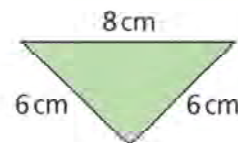
- 3 Are these two triangles congruent?

Give a reason to support your answer.



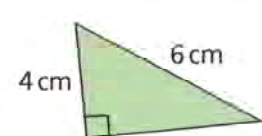
- 4 Are these two triangles congruent?

Give a reason to support your answer.



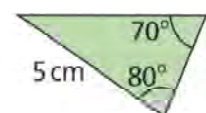
- 5 Are these two triangles congruent?

Give a reason to support your answer.



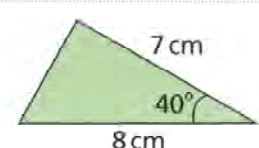
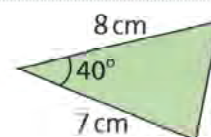
- 6 Are these two triangles congruent?

Give a reason to support your answer.



- 7 Are these two triangles congruent?

Give a reason to support your answer.

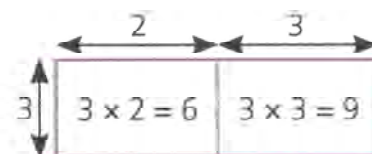


Expanding two or more binomials

The distributive law

The **distributive law** says that the **product** of two numbers multiplied together is the same as the sum of the product of those numbers split into groups, or **partitioned**.

For example, $3(2 + 3)$ can be shown using an area model. Imagine a rectangle split into two sections. To find the area of the whole rectangle, add up the area of each section.



The distributive law can be shown without visual aids by **expanding the bracket**.

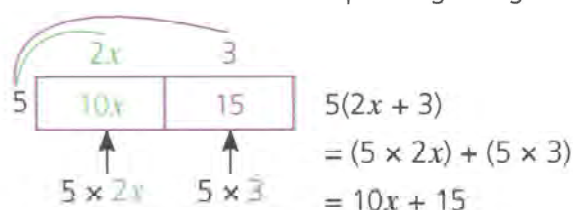
$$3(2 + 3) = (3 \times 2) + (3 \times 3) \\ = 6 + 9 = 15$$

$$\text{Whole area} = 6 + 9 \\ = 15$$

Using an area model (the grid method)

The distributive law also applies when there are variables inside or outside the brackets.

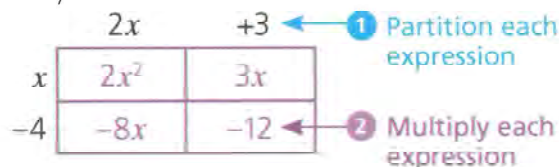
Recall the area method for expanding a single bracket.



Keep any negative sign with the number when partitioning the brackets.

This model can be extended to finding the **product** of two **binomials** (expressions with two terms):

$$(2x + 3)(x - 4)$$



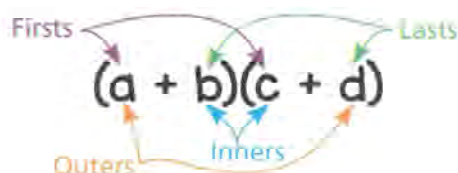
3 Simplify:

$$2x^2 + 3x - 8x - 12 \\ 2x^2 - 5x - 12 \\ (2x + 3)(x - 4) = 2x^2 - 5x - 12$$

Using the FOIL method

Two brackets can be expanded using a method called FOIL.

- First: multiply the first terms of both brackets
- Outer: multiply the outer terms of both brackets
- Inner: multiply the inner terms of both brackets
- Last: multiply the last terms of both brackets



Find the product of $(2x + 3)(x - 4)$

$$(2x + 3)(x - 4) = (2x \times x) + (2x \times -4) + (3 \times x) + (3 \times -4) \\ = 2x^2 - 8x + 3x - 12 \\ = 2x^2 - 5x - 12$$

Make sure you multiply **every** term in the first bracket by **every** term in the second bracket.

Expanding more than two binomials

Both the area model and FOIL can be extended to multiplying more than two brackets. Expand any two brackets first, simplify, then multiply the new expression by the third bracket.

Find the product of $(2x + 3)(x - 4)(x - 3)$

$$(2x + 3)(x - 4) = 2x^2 - 5x - 12 \quad \text{As shown above.}$$

Now multiply $(2x^2 - 5x - 12)(x - 3)$ Multiply the product of the first two brackets by the third bracket.

	$2x^2$	$-5x$	-12
x	$2x^2 \times x = 2x^3$	$-5x \times x = -5x^2$	$-12 \times x = -12x$
-3	$2x^2 \times -3 = -6x^2$	$-5x \times -3 = 15x$	$-12 \times -3 = 36$

$$= 2x^3 - 6x^2 - 5x^2 + 15x - 12x + 36 \\ = 2x^3 - 11x^2 + 3x + 36$$

FOIL method

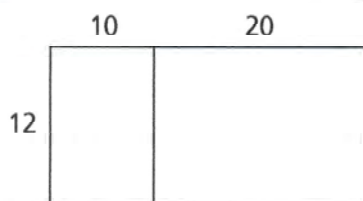
$$(2x^2 - 5x - 12)(x - 3) = \\ (2x^2 \times x) + (2x^2 \times -3) + (-5x \times x) + \\ (-5x \times -3) + (-12 \times x) + (-12 \times -3)$$

You could first multiply $(x - 4)(x - 3)$, then multiply by $(2x + 3)$, and get the same answer.

Expanding two or more binomials

The distributive law

- 1 Find the area of this rectangle using the distributive law.



$$(\quad \times \quad) + (\quad \times \quad) = \quad \times \quad$$

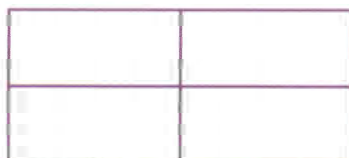
$$\quad + \quad = \quad$$

Using an area model (the grid method)

- 2 Use an area model to find the product of:

a) $(x - 4)(4x + 3)$

b) $(2n - 1)(3n - 2)$



Using the FOIL method

- 3 Use the FOIL method to find the product of:

a) $(4y + 2)(y - 5)$

b) $(2k + 3)(3k + 5)$

Expanding more than two binomials

- 4 Find the product of $(x - 4)(4x + 3)(x - 2)$ using any method. You may wish to use your answer from question 2a) to help you.