

# Adding fractions

## Adding fractions with the same denominator

To add fractions with the same denominator, simply add the numerators. The answer may be greater than 1.

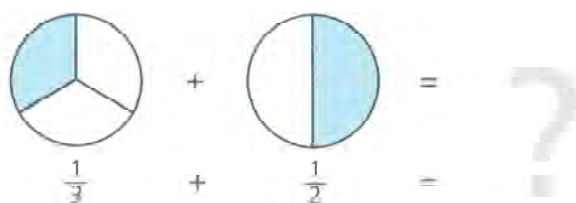
$$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$$

$$\frac{5}{7} + \frac{4}{7} = \frac{5+4}{7} = \frac{9}{7} = 1\frac{2}{7}$$

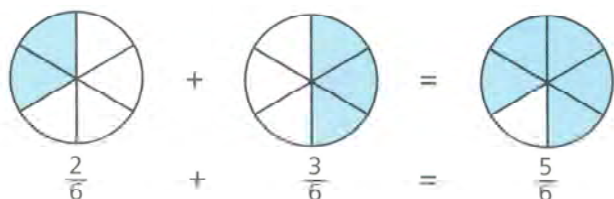
## Adding fractions with different denominators

To add fractions, the denominators must first be the same.

You can't add  $\frac{1}{3} + \frac{1}{2}$  directly.



However, using a common denominator of 6:



$$\frac{3}{4} + \frac{6}{18} = ?$$

A common multiple of 4 and 18 is 36.

$$\frac{3}{4} = \frac{27}{36}$$

(Diagram showing 3/4 multiplied by 9 to get 27/36)

$$\frac{6}{18} = \frac{12}{36}$$

(Diagram showing 6/18 multiplied by 2 to get 12/36)

$$\begin{aligned} \frac{3}{4} + \frac{6}{18} &= \frac{27}{36} + \frac{12}{36} = \frac{27+12}{36} \\ &= \frac{39}{36} = \frac{13}{12} = 1\frac{1}{12} \end{aligned}$$

Find a common denominator and use equivalent fractions to add the fractions.

## Adding mixed numbers

**Method 1:** Partition the mixed numbers into their integer and fraction components, then add them.

**Method 2:** Convert the mixed numbers to improper fractions, then add them, and finally convert the answer back to a mixed number if needed.

Just as when adding proper fractions, the fractions must have a common denominator.

If the answer has an improper fraction in it as well, convert that to a mixed number and add it to the integer component.

$$1\frac{3}{5} + 2\frac{1}{8} = ?$$

**Method 1:**

$$1 + 2 + \frac{3}{5} + \frac{1}{8}$$

First partition the integers and fractions.

$$1 + 2 = 3$$

Add the integers.

$$\frac{3}{5} = \frac{24}{40}$$

(Diagram showing 3/5 multiplied by 8 to get 24/40)

$$\frac{1}{8} = \frac{5}{40}$$

(Diagram showing 1/8 multiplied by 5 to get 5/40)

Now add the fractions. Find a common denominator. 5 and 8 have a common multiple of 40.

$$\frac{24}{40} + \frac{5}{40} = \frac{29}{40}$$

$$\text{Therefore, } 1\frac{3}{5} + 2\frac{1}{8} = 3\frac{29}{40}$$

**Method 2:**

Convert the mixed numbers to improper fractions.

$$1\frac{3}{5} = \frac{(5 \times 1) + 3}{5} = \frac{8}{5}$$

$$2\frac{1}{8} = \frac{(8 \times 2) + 1}{8} = \frac{17}{8}$$

$$\frac{8}{5} = \frac{64}{40}$$

(Diagram showing 8/5 multiplied by 8 to get 64/40)

$$\frac{17}{8} = \frac{85}{40}$$

(Diagram showing 17/8 multiplied by 5 to get 85/40)

Find a common denominator.

$$\frac{64}{40} + \frac{85}{40} = \frac{149}{40}$$

$$149 \div 40 = 3 \text{ r}29, \text{ so } \frac{149}{40} = 3\frac{29}{40}$$

$$1\frac{3}{5} + 2\frac{1}{8} = 3\frac{29}{40}$$

Convert the improper fraction to a mixed number.

# Adding fractions

## Adding fractions with the same denominator

- 1 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a)  $\frac{4}{8} + \frac{3}{8}$

b)  $\frac{12}{20} + \frac{15}{20}$

## Adding fractions with different denominators

- 2 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a)  $\frac{3}{12} + \frac{5}{8}$

b)  $\frac{7}{9} + \frac{3}{4}$

## Adding mixed numbers

- 3 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a)  $7\frac{2}{5} + 3\frac{1}{8}$

b)  $5\frac{4}{5} + 1\frac{3}{8}$

# Dividing fractions

## Dividing an integer by a fraction

Division is another way of asking how many times one quantity fits into another. Dividing an integer by a fraction is asking how many times that fraction fits into the integer.

Work out  $4 \div \frac{2}{3}$

To find  $4 \div \frac{2}{3}$ , a bar with a value of 4 can be divided into parts representing  $\frac{2}{3}$

| 4             |               |               |               |               |               |               |               |               |               |               |               |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1             |               |               | 1             |               |               | 1             |               |               | 1             |               |               |
| $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

There are 6 sections showing  $\frac{2}{3}$

Therefore,  $4 \div \frac{2}{3} = 6$

Without a bar model, you can carry out division calculations involving fractions by using the **reciprocal method**. This involves keeping the first number the same, using the reciprocal of the second number, and changing the division to multiplication.

**K** keep the first number the same  
**F** lip the second number  
**C** hange  $\div$  to  $\times$

To find the reciprocal of a fraction, the numerator becomes the denominator, and the denominator becomes the numerator. Think of the reciprocal of a fraction as the fraction flipped upside down.

Work out  $6 \div \frac{2}{3}$

$$6 \div \frac{2}{3} = 6 \times \frac{3}{2}$$

Keep the first value the same      Change  $\div$  to  $\times$       Flip the second value

$$= \frac{6 \times 3}{2} = \frac{18}{2} = 9$$

## Dividing a fraction by a fraction

Dividing a fraction by a fraction means asking how many times one fraction fits into another.

To divide a fraction by a fraction, use the KFC method.

$\frac{3}{4} \div \frac{1}{2}$  means how many halves are in three quarters.

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$$

Keep the first value the same      Change  $\div$  to  $\times$       Flip the second value

$$= \frac{3 \times 2}{4 \times 1}$$

$$= \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

Work out  $\frac{2}{5} \div \frac{7}{9}$

$$\frac{2}{5} \div \frac{7}{9} = \frac{2}{5} \times \frac{9}{7}$$

$$= \frac{2 \times 9}{5 \times 7}$$

$$= \frac{18}{35}$$

You can remember the reciprocal method by KFC: Keep, Flip, Change

## Dividing with mixed numbers

If the calculation involves mixed numbers, first convert the mixed number to an improper fraction. Then use the reciprocal method to divide.

Work out  $2\frac{1}{3} \div \frac{5}{9}$

$$2\frac{1}{3} = \frac{(3 \times 2) + 1}{3} = \frac{7}{3}$$

Convert  $2\frac{1}{3}$  to an improper fraction.

$$\frac{7}{3} \div \frac{5}{9} = \frac{7}{3} \times \frac{9}{5}$$

Use the reciprocal method.

$$= \frac{7 \times 9}{3 \times 5} = \frac{63}{15}$$

$$\frac{63}{15} = 4\frac{3}{15} = 4\frac{1}{5}$$

Simplify the answer if possible.

# Dividing fractions

## Dividing an integer by a fraction

1 Divide these integers and fractions.

a)  $9 \div \frac{2}{3}$

b)  $6 \div \frac{5}{6}$

## Dividing a fraction by a fraction

2 Divide these fractions.

a)  $\frac{1}{6} \div \frac{3}{8}$

b)  $\frac{1}{9} \div \frac{9}{4}$

c)  $\frac{1}{2} \div \frac{2}{3}$

## Dividing with mixed numbers

3 Divide these fractions and mixed numbers.

a)  $3\frac{2}{7} \div \frac{1}{2}$

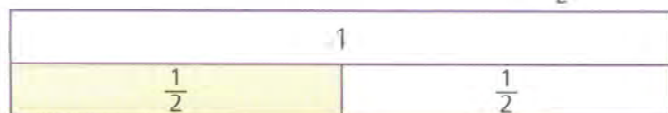
b)  $5\frac{2}{3} \div \frac{1}{3}$

# Multiplying fractions

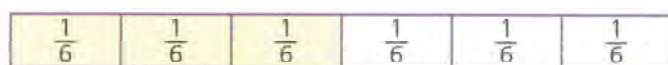
## Multiplying fractions by fractions

Multiplying by a fraction means finding a fraction of an amount. Multiplying two fractions means finding a fraction of the fraction.

$\frac{2}{3} \times \frac{1}{2}$  means finding  $\frac{2}{3}$  of  $\frac{1}{2}$ . This can be shown using a bar model. Here is a bar showing  $\frac{1}{2}$ :



Now divide both parts of the bar into thirds. The bar now has 6 parts with half of the bar shaded.



Then find  $\frac{2}{3}$  of the shaded part.



$\frac{2}{3}$  of  $\frac{1}{2}$  of the bar is  $\frac{2}{6}$  of the bar, or  $\frac{1}{3}$ .

To multiply fractions without a bar model:

- multiply the numerator by the numerator
- multiply the denominator by the denominator
- simplify the fraction.

Work out  $\frac{2}{3} \times \frac{6}{8}$

$$\frac{2}{3} \times \frac{6}{8} = \frac{2 \times 6}{3 \times 8} = \frac{12}{24} = \frac{1}{2}$$

To multiply mixed numbers, first convert them to improper fractions.

Work out  $2\frac{3}{4} \times 1\frac{2}{5}$

$$2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{11}{4}$$

$$1\frac{2}{5} = \frac{(5 \times 1) + 2}{5} = \frac{7}{5}$$

$$\frac{11}{4} \times \frac{7}{5} = \frac{11 \times 7}{4 \times 5}$$

$$= \frac{77}{20} = 3\frac{17}{20}$$

Then multiply the improper fractions and simplify the answer.

To do these calculations on a calculator, find the mixed number button and enter the calculation.

**Multiply the numerator by the numerator and the denominator by the denominator.**

## Multiplying fractions by integers

To multiply a fraction by an integer, remember that an integer can be written as a fraction by writing the integer as the numerator and writing the denominator as 1.

Then to multiply the fraction and integer, multiply the numerators and the denominators as before.

$$5 \times \frac{2}{3} = \frac{5}{1} \times \frac{2}{3} \quad 5 = \frac{5}{1}$$

$$= \frac{5 \times 2}{1 \times 3} = \frac{10}{3} = 3\frac{1}{3}$$

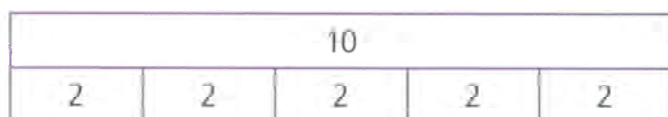
The step of writing the integer as a fraction can be left out. Multiply the integer by the numerator and keep the denominator the same (because the denominator was being multiplied by 1).

**Multiply the integer by the numerator and keep the denominator the same.**

## Finding a fraction of an amount

Finding a fraction of an amount means to multiply that amount by a fraction.

$\frac{3}{5}$  of £10 can be shown using a bar model.



$\frac{3}{5}$  of 10 = 6

Find  $\frac{4}{5}$  of £20

£20 ÷ 5 = £4 so  $\frac{1}{5}$  of £20 is £4 To find  $\frac{4}{5}$ , first find  $\frac{1}{5}$

Then  $\frac{4}{5}$  of £20 is 4 × £4 = £16

Alternatively, multiply the amount by the numerator and divide by the denominator.

$$\frac{4}{5} \times 20 = \frac{(4 \times 20)}{5} = \frac{80}{5}$$

80 ÷ 5 = 16, so  $\frac{4}{5}$  of £20 is £16

# Multiplying fractions

## Multiplying fractions by fractions

1 Multiply these fractions. Simplify your answers.

a)  $\frac{7}{9} \times \frac{5}{8}$

.....

b)  $\frac{2}{5} \times \frac{3}{4}$

.....

c)  $1\frac{5}{6} \times 2\frac{2}{5}$

.....

## Multiplying fractions by integers

2 Multiply these fractions and integers. Simplify your answers.

a)  $3 \times \frac{5}{8}$

.....

b)  $9 \times \frac{3}{4}$

.....

c)  $8 \times \frac{2}{5}$

.....

## Finding a fraction of an amount

3 Find these fractions of amounts.

a)  $\frac{4}{5}$  of £200

£ .....

b)  $\frac{2}{7}$  of 49 sweets

..... sweets

c)  $\frac{5}{9}$  of 54 cm

..... cm

## Powers

A **power** is a shorter way of writing repeated multiplication of the same number.

A power is also sometimes called an **index** or **exponent**. The number being multiplied is called the **base**.

Power (or index)

$$8^5 = 8 \times 8 \times 8 \times 8 \times 8$$

Base

In words, this is said '8 to the power of 5'.

$$3 \times 3 = 3^2$$

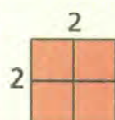
'3 squared'

$$3 \times 3 \times 3 = 3^3$$

'3 cubed'

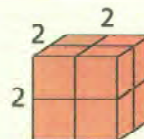
$$3 \times 3 \times 3 \times 3 = 3^4$$

'3 to the power of 4'



$$2 \times 2 = 4$$

$$2^2 = 4$$



$$2 \times 2 \times 2 = 8$$

$$2^3 = 8$$

A number raised to the power of 2 is called **squared**. A number raised to the power of 3 is called **cubed**.

## Roots

A **root** is the **inverse** of a power. In other words, it is the base: the number that is being multiplied by itself. For example,  $\sqrt{4}$  means 'What number multiplied by itself gives 4?'

$$3^3 = 27, \text{ so } \sqrt[3]{27} = 3$$

In words: 'the cube root of 27 is 3'

$$3^5 = 243, \text{ so } \sqrt[5]{243} = 3$$

In words: 'the fifth root of 243 is 3'

The square root is the number that is squared to make a given number.

## Estimating square roots

You can estimate a square root by knowing the square roots of nearby numbers.

The square root of 96 is between 9 and 10.

$$9^2 = 81, \text{ so } \sqrt{81} = 9$$

$$10^2 = 100, \text{ so } \sqrt{100} = 10$$

Since 96 is closer to 100 than to 81, the square root will be between 9.5 and 9.9 to 1 decimal place.

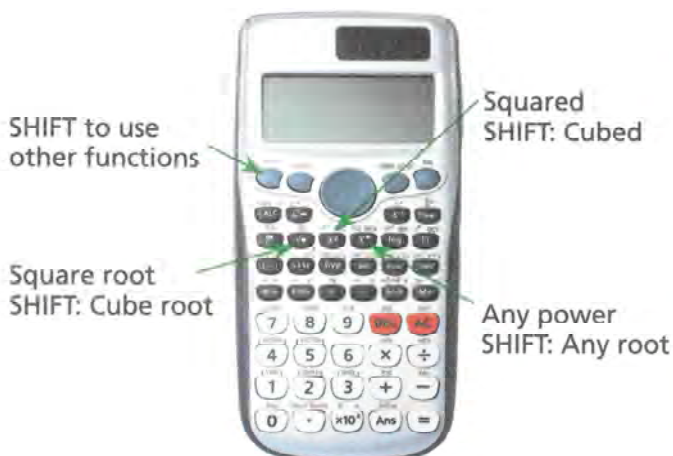
Using a calculator, we see that  $\sqrt{96} = 9.8$  to 1 d.p.

You should know all the square numbers up to 144 from your times tables.

## Using a calculator

You can use a calculator to find other powers and roots. You will often need to use the **SHIFT** or **2<sup>nd</sup>F** buttons to access other functions.

The buttons to find any power and any root may look like  $x^y$  and  $\sqrt[y]{x}$ .



$$3^8 = 6561$$

$$3 \text{ } x^y \text{ } 8 \text{ } =$$

Check the order you must push the buttons on your calculator. You may need to push  $x^y$  first.

$$\sqrt[3]{1728} = 12$$

$$\text{SHIFT } \sqrt[3]{\square} \text{ } 1 \text{ } 7 \text{ } 2 \text{ } 8 \text{ } =$$

Again, check the order you must push the buttons on your own calculator.

Find the buttons on your own calculator for powers and roots and learn how to use them.

## Powers

1 Write the following as powers.

a)  $5 \times 5 \times 5 \times 5 \times 5 \times 5$  .....

b)  $9 \times 9 \times 9$  .....

c)  $2 \times 2 \times 2 \times 2$  .....

2 Write the following powers as multiplications.

a)  $3^8$  .....

b)  $4^5$  .....

c)  $10^4$  .....

3 Find the value of:

a)  $7^2$  .....

b)  $2^4$  .....

## Roots

4 Find the square roots.

a)  $\sqrt{36}$  .....

b)  $\sqrt{121}$  .....

5 Find the cube roots.

a)  $\sqrt[3]{8}$  .....

b)  $\sqrt[3]{64}$  .....

## Estimating square roots

6 Estimate the square roots (to 1 decimal place).

a)  $\sqrt{135}$  .....

b)  $\sqrt{200}$  .....

c)  $\sqrt{78}$  .....

## Using a calculator

7 Use a calculator to find the value of:

a)  $2^6$  .....

b)  $3^5$  .....

c)  $\sqrt{256}$  .....

d)  $\sqrt[3]{3375}$  .....

# Subtracting fractions

## Subtracting fractions with the same denominator

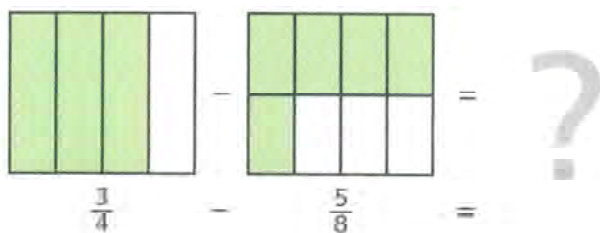
Subtracting fractions is the same process as adding fractions, just using the operation of subtraction rather than addition.

$$\frac{8}{12} - \frac{5}{12} = \frac{8-5}{12} = \frac{3}{12} = \frac{1}{4}$$

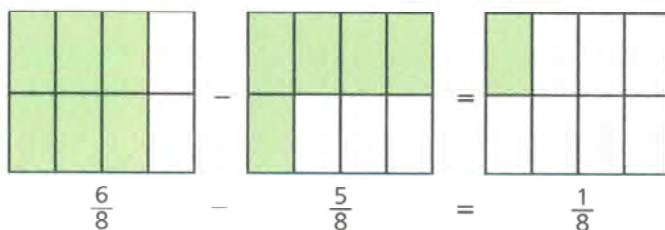
## Subtracting fractions with different denominators

To subtract fractions, the denominators must first be the same. If they are different, find a common denominator and use equivalent fractions.

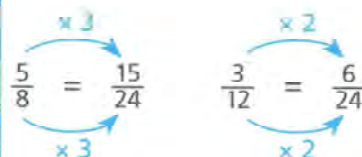
You can't subtract  $\frac{3}{4} - \frac{5}{8}$  directly.



However, using a common denominator of 8:



Work out  $\frac{5}{8} - \frac{3}{12}$



A common multiple of 8 and 12 is 24.

$$\begin{aligned} \frac{5}{8} - \frac{3}{12} &= \frac{15}{24} - \frac{6}{24} \\ &= \frac{15-6}{24} \\ &= \frac{9}{24} = \frac{3}{8} \end{aligned}$$

Find a common denominator and use equivalent fractions to subtract fractions.

## Subtracting mixed numbers

**Method 1:** Partition the mixed numbers into their integer and fraction components, then subtract them.

**Method 2:** Convert the mixed numbers to improper fractions, then subtract them, and finally convert the answer back to a mixed number if needed.

You may find one method easier to use than the other. Learning how to use the fraction buttons on a calculator will help.

Work out  $8\frac{1}{2} - 4\frac{5}{8}$

**Method 1:**

$$8\frac{1}{2} - 4\frac{5}{8} = 8 + \frac{1}{2} - 4 - \frac{5}{8}$$

Partition the mixed numbers.

Keep track of the negative signs. Don't forget you are subtracting the integer and the fraction!

$$8 - 4 = 4 \quad \text{Subtract the integers.}$$

$$\frac{1}{2} = \frac{4}{8}$$

Subtract the fractions. Find a common denominator, 8.

$$\frac{4}{8} - \frac{5}{8} = \frac{4-5}{8} = -\frac{1}{8}$$

$$8\frac{1}{2} - 4\frac{5}{8} = 4 - \frac{1}{8}$$

This is where it gets tricky.

The answer is  $\frac{1}{8}$  less than 4, which is  $3\frac{7}{8}$

**Method 2:**

$$8\frac{1}{2} = 8\frac{4}{8} \quad \text{Find a common denominator, 8.}$$

$$8\frac{4}{8} = \frac{(8 \times 8) + 4}{8} = \frac{68}{8}$$

$$4\frac{5}{8} = \frac{(4 \times 8) + 5}{8} = \frac{37}{8}$$

Convert to improper fractions and subtract.

$$\frac{68}{8} - \frac{37}{8} = \frac{31}{8} = 3\frac{7}{8}$$

# Subtracting fractions

## Subtracting fractions with the same denominator

1 Subtract these fractions.

a)  $\frac{5}{8} - \frac{2}{8}$

.....

b)  $\frac{13}{15} - \frac{8}{15}$

.....

## Subtracting fractions with different denominators

2 Subtract these fractions.

a)  $\frac{6}{7} - \frac{3}{14}$

.....

b)  $\frac{5}{7} - \frac{4}{9}$

.....

## Subtracting mixed numbers

3 Subtract these fractions. Give your answer as a mixed number when the answer is greater than 1.

a)  $4\frac{2}{3} - 2\frac{1}{8}$

.....

b)  $2\frac{2}{5} - 1\frac{5}{7}$

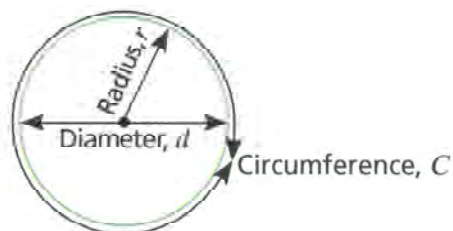
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## Circumference of a circle

Circumference of a circle = perimeter of circle

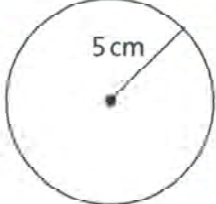
Diameter =  $2 \times$  radius

$$d = 2r$$



**Circumference of a circle:**  $C = 2\pi r$  or  $C = \pi d$

Work out the circumference of each circle. Give your answers to 1 decimal place.

a)  You know the radius, so use  $C = 2\pi r$

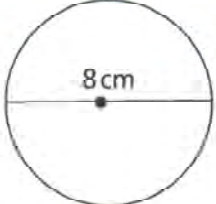
$$C = 2\pi r$$

$$= 2 \times \pi \times 5$$

Use the  $\pi$  key on your calculator.

$$= 31.41592\dots$$

$$C = 31.4 \text{ cm (1 d.p.)}$$

b)  You know the diameter, so use  $C = \pi d$

$$C = \pi d$$

$$= \pi \times 8$$

$$= 25.13274\dots$$

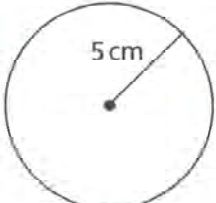
$$C = 25.1 \text{ cm (1 d.p.)}$$

## Area of a circle

The area of a circle is the space inside the circumference.

**Area of a circle:**  $A = \pi r^2$

Work out the area of each circle. Give your answers to 1 decimal place.

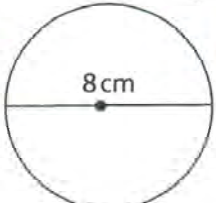
a)   $A = \pi r^2$

$$= \pi \times 5^2$$

Use the  $\pi$  key on your calculator.

$$= 78.5398\dots$$

$$A = 78.5 \text{ cm}^2 \text{ (1 d.p.)}$$

b)  You know the diameter, so find the radius first.

$$r = 8 \div 2 = 4$$

$$A = \pi r^2$$

$$= \pi \times 4^2$$

$$= 50.2654\dots$$

$$A = 50.3 \text{ cm}^2 \text{ (1 d.p.)}$$

## Area and circumference problems

Remember which formula is which.

**Area**

$A = \pi r^2$  <sup>2 in formula and units</sup> measured in  $\text{cm}^2$  (or  $\text{km}^2$  or  $\text{m}^2$  or  $\text{mm}^2$ )

**Circumference**

$C = 2\pi r$  or  $C = \pi d$  <sup>No 2 in formula or units</sup> measured in cm (or km or m or mm)

A circle has circumference 36 cm. Work out its area. Give your answer to 1 decimal place.

$2\pi r = 36$  Use  $C = 2\pi r$  to write an equation.

$\div 2\pi$   $\frac{2\pi r = 36}{\div 2\pi} \Rightarrow r = \frac{36}{2\pi}$  Solve the equation to find  $r$ .

$r = 5.7295\dots \text{ cm}$  Use a calculator. Keep several decimal places in your value for  $r$ .

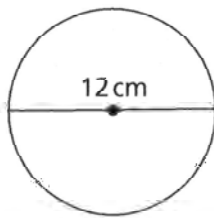
$A = \pi r^2$   
 $= \pi \times 5.7295^2$  Use your value of  $r$ .

$= 103.1295\dots$   
 $A = 103.1 \text{ cm}^2 \text{ (1 d.p.)}$  Round to 1 decimal place.

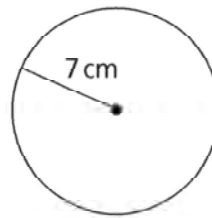
## Circumference of a circle

- 1 Work out the circumference of each circle. Give your answers to 1 decimal place.

a)



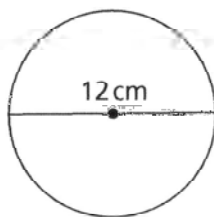
b)



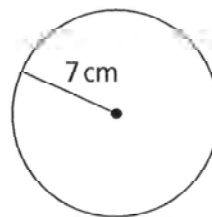
## Area of a circle

- 2 Work out the area of each circle. Give your answers to 1 decimal place.

a)



b)



## Area and circumference problems

- 3 A circle has circumference 50 cm.

Work out its radius. Give your answer to 1 decimal place.

- 4 A circle has circumference 72 cm.

Work out its diameter. Give your answer to 1 decimal place.

- 5 A circle has area  $48\text{ cm}^2$ .

Giving your answers to 1 decimal place, work out:

a) its radius

b) its diameter

c) its circumference



# Direct proportion

## Direct proportion word problems

1 If 3 pencils cost 75p, how much do 6 pencils cost?

.....

2 If 4 lollipops cost 80p, how much do 2 lollipops cost?

.....

3 Jeremiah can type 60 words in 1 minute.

How many words can Jeremiah type in 3 minutes?

.....

4 An electrician works for 3 hours and gets paid £150.

How much will the electrician be paid for working for 5 hours?

.....

5 If 2 pencils cost 44p, how much will 9 pencils cost?

.....

6 A car travels 120 miles in 4 hours at a steady speed.

How far does the car travel in 7 hours?

.....

## Direct proportion formula

7  $A$  is directly proportional to  $B$  and when  $A = 24$ ,  $B = 4$ .

Find the value of  $A$  when  $B$  is 9.

.....

8  $X$  is directly proportional to  $Y$  and when  $X = 35$ ,  $Y = 7$ .

Find the value of  $X$  when  $Y$  is 9.

.....

9  $P$  is directly proportional to  $Q$  and when  $P = 60$ ,  $Q = 120$ .

Find the value of  $P$  when  $Q$  is 100.

.....