ORGANISE

6 Adding fractions

Adding fractions with the same denominator

To add fractions with the same denominator, simply add the numerators. The answer may be greater than 1.

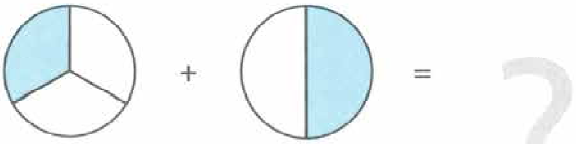
$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$

$\frac{5}{7} + \frac{4}{7} = \frac{5+4}{7} = \frac{9}{7} = 1\frac{2}{7}$

Adding fractions with different denominators

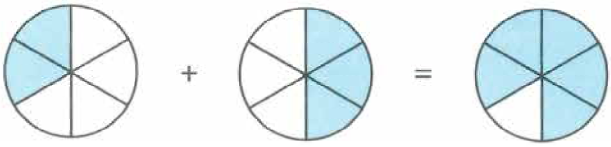
To add fractions, the denominators must first be the same.

You can't add $\frac{1}{3} + \frac{1}{2}$ directly.



$\frac{1}{3} + \frac{1}{2} =$?

However, using a common denominator of 6:



$\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

$\frac{3}{4} + \frac{6}{18} = ?$

A common multiple of 4 and 18 is 36.

$\frac{3}{4} = \frac{27}{36}$ (× 9) $\frac{6}{18} = \frac{12}{36}$ (× 2)

$\frac{3}{4} + \frac{6}{18} = \frac{27}{36} + \frac{12}{36} = \frac{27+12}{36}$

$= \frac{39}{36} = \frac{13}{12} = 1\frac{1}{12}$

Find a common denominator and use equivalent fractions to add the fractions.

Adding mixed numbers

Method 1: Partition the mixed numbers into their integer and fraction components, then add them.

Method 2: Convert the mixed numbers to improper fractions, then add them, and finally convert the answer back to a mixed number if needed.

Just as when adding proper fractions, the fractions must have a common denominator.

If the answer has an improper fraction in it as well, convert that to a mixed number and add it to the integer component.

$1\frac{3}{5} + 2\frac{1}{8} = ?$

Method 1:

$1 + 2 + \frac{3}{5} + \frac{1}{8}$ First partition the integers and fractions.

$1 + 2 = 3$ Add the integers.

$\frac{3}{5} = \frac{24}{40}$ (× 8) $\frac{1}{8} = \frac{5}{40}$ (× 5)

Now add the fractions. Find a common denominator. 5 and 8 have a common multiple of 40.

$\frac{24}{40} + \frac{5}{40} = \frac{29}{40}$

Therefore, $1\frac{3}{5} + 2\frac{1}{8} = 3\frac{29}{40}$

Method 2:

Convert the mixed numbers to improper fractions.

$1\frac{3}{5} = \frac{(5 \times 1) + 3}{5} = \frac{8}{5}$ $2\frac{1}{8} = \frac{(8 \times 2) + 1}{8} = \frac{17}{8}$

$\frac{8}{5} = \frac{64}{40}$ (× 8) $\frac{17}{8} = \frac{85}{40}$ (× 5) Find a common denominator.

$\frac{64}{40} + \frac{85}{40} = \frac{149}{40}$

$149 \div 40 = 3$ r29, so $\frac{149}{40} = 3\frac{29}{40}$ Convert the improper fraction to a mixed number.

$1\frac{3}{5} + 2\frac{1}{8} = 3\frac{29}{40}$

RETRIEVE

6 Adding fractions

Adding fractions with the same denominator

1 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a) $\frac{4}{8} + \frac{3}{8}$

b) $\frac{12}{20} + \frac{15}{20}$

Adding fractions with different denominators

2 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a) $\frac{3}{12} + \frac{5}{8}$

b) $\frac{7}{9} + \frac{3}{4}$

Adding mixed numbers

3 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a) $7\frac{2}{5} + 3\frac{1}{8}$

b) $5\frac{4}{5} + 1\frac{3}{8}$

ORGANISE

6 Subtracting fractions

Subtracting fractions with the same denominator

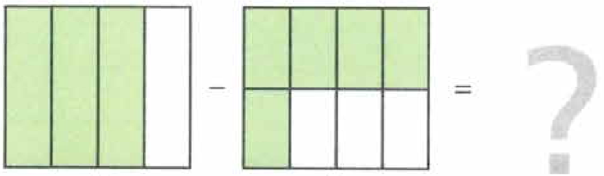
Subtracting fractions is the same process as adding fractions, just using the operation of subtraction rather than addition.

$$\frac{8}{12} - \frac{5}{12} = \frac{8-5}{12} = \frac{3}{12} = \frac{1}{4}$$

Subtracting fractions with different denominators

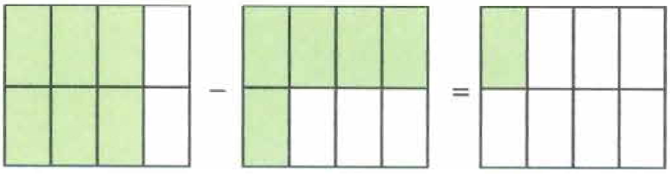
To subtract fractions, the denominators must first be the same. If they are different, find a common denominator and use equivalent fractions.

You can't subtract $\frac{3}{4} - \frac{5}{8}$ directly.



$\frac{3}{4}$ − $\frac{5}{8}$ =

However, using a common denominator of 8:



$\frac{6}{8}$ − $\frac{5}{8}$ = $\frac{1}{8}$

Work out $\frac{5}{8} - \frac{3}{12}$

$\frac{5}{8} = \frac{15}{24}$ (× 3 top and bottom) $\frac{3}{12} = \frac{6}{24}$ (× 2 top and bottom)

A common multiple of 8 and 12 is 24.

$$\frac{5}{8} - \frac{3}{12} = \frac{15}{24} - \frac{6}{24}$$
$$= \frac{15-6}{24}$$
$$= \frac{9}{24} = \frac{3}{8}$$

Find a common denominator and use equivalent fractions to subtract fractions.

Subtracting mixed numbers

Method 1: Partition the mixed numbers into their integer and fraction components, then subtract them.

Method 2: Convert the mixed numbers to improper fractions, then subtract them, and finally convert the answer back to a mixed number if needed.

You may find one method easier to use than the other. Learning how to use the fraction buttons on a calculator will help.

Work out $8\frac{1}{2} - 4\frac{5}{8}$

Method 1:

$8\frac{1}{2} - 4\frac{5}{8} = 8 + \frac{1}{2} - 4 - \frac{5}{8}$ Partition the mixed numbers.

Keep track of the negative signs. Don't forget you are subtracting the integer and the fraction!

$8 - 4 = 4$ Subtract the integers.

$\frac{1}{2} = \frac{4}{8}$

$\frac{4}{8} - \frac{5}{8} = \frac{4-5}{8} = -\frac{1}{8}$ Subtract the fractions. Find a common denominator, 8.

$8\frac{1}{2} - 4\frac{5}{8} = 4 - \frac{1}{8}$ This is where it gets tricky.

The answer is $\frac{1}{8}$ less than 4, which is $3\frac{7}{8}$

Method 2:

$8\frac{1}{2} = 8\frac{4}{8}$ Find a common denominator, 8.

$8\frac{4}{8} = \frac{(8 \times 8) + 4}{8} = \frac{68}{8}$

$4\frac{5}{8} = \frac{(8 \times 4) + 5}{8} = \frac{37}{8}$

Convert to improper fractions and subtract.

$\frac{68}{8} - \frac{37}{8} = \frac{31}{8} = 3\frac{7}{8}$

RETRIEVE

6 Subtracting fractions

Subtracting fractions with the same denominator

1. Subtract these fractions.

a) $\frac{5}{8} - \frac{2}{8}$

....

b) $\frac{13}{15} - \frac{8}{15}$

....

Subtracting fractions with different denominators

2. Subtract these fractions.

a) $\frac{6}{7} - \frac{3}{14}$

....

b) $\frac{5}{7} - \frac{4}{9}$

....

Subtracting mixed numbers

3. Subtract these fractions. Give your answer as a mixed number when the answer is greater than 1.

a) $4\frac{2}{3} - 2\frac{1}{8}$

....

b) $2\frac{2}{5} - 1\frac{5}{7}$

....

6 Multiplying fractions

Multiplying fractions by fractions

Multiplying by a fraction means finding a fraction of an amount. Multiplying two fractions means finding a fraction of the fraction.

$\frac{2}{3} \times \frac{1}{2}$ means finding $\frac{2}{3}$ of $\frac{1}{2}$. This can be shown using a bar model. Here is a bar showing $\frac{1}{2}$:

1	
$\frac{1}{2}$	$\frac{1}{2}$

Now divide both parts of the bar into thirds. The bar now has 6 parts with half of the bar shaded.

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Then find $\frac{2}{3}$ of the shaded part.

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{2}{3}$ of $\frac{1}{2}$ of the bar is $\frac{2}{6}$ of the bar, or $\frac{1}{3}$

To multiply fractions without a bar model:

- multiply the numerator by the numerator
- multiply the denominator by the denominator
- simplify the fraction.

Work out $\frac{2}{3} \times \frac{6}{8}$

$\frac{2}{3} \times \frac{6}{8} = \frac{2 \times 6}{3 \times 8} = \frac{12}{24} = \frac{1}{2}$

To multiply mixed numbers, first convert them to improper fractions.

Work out $2\frac{3}{4} \times 1\frac{2}{5}$

$2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{11}{4}$ $1\frac{2}{5} = \frac{(5 \times 1) + 2}{5} = \frac{7}{5}$

$\frac{11}{4} \times \frac{7}{5} = \frac{11 \times 7}{4 \times 5}$

$= \frac{77}{20} = 3\frac{17}{20}$

Then multiply the improper fractions and simplify the answer.

To do these calculations on a calculator, find the mixed number button and enter the calculation.

Multiply the numerator by the numerator and the denominator by the denominator.

Multiplying fractions by integers

To multiply a fraction by an integer, remember that an integer can be written as a fraction by writing the integer as the numerator and writing the denominator as 1.

Then to multiply the fraction and integer, multiply the numerators and the denominators as before.

$5 \times \frac{2}{3} = \frac{5}{1} \times \frac{2}{3}$ $5 = \frac{5}{1}$

$= \frac{5 \times 2}{1 \times 3} = \frac{10}{3} = 3\frac{1}{3}$

The step of writing the integer as a fraction can be left out. Multiply the integer by the numerator and keep the denominator the same (because the denominator was being multiplied by 1).

Multiply the integer by the numerator and keep the denominator the same.

Finding a fraction of an amount

Finding a fraction of an amount means to multiply that amount by a fraction.

$\frac{3}{5}$ of £10 can be shown using a bar model.

10				
2	2	2	2	2

$\frac{3}{5}$ of 10 = 6

Find $\frac{4}{5}$ of £20

£20 ÷ 5 = £4 so $\frac{1}{5}$ of £20 is £4 To find $\frac{4}{5}$, first find $\frac{1}{5}$

Then $\frac{4}{5}$ of £20 is 4 × £4 = £16

Alternatively, multiply the amount by the numerator and divide by the denominator.

$\frac{4}{5} \times 20 = \frac{(4 \times 20)}{5} = \frac{80}{5}$

80 ÷ 5 = 16, so $\frac{4}{5}$ of £20 is £16

RETRIEVE

6 Multiplying fractions

Multiplying fractions by fractions

1. Multiply these fractions. Simplify your answers.

a) $\frac{7}{9} \times \frac{5}{8}$

....

b) $\frac{2}{5} \times \frac{3}{4}$

....

c) $1\frac{5}{6} \times 2\frac{2}{5}$

....

Multiplying fractions by integers

2. Multiply these fractions and integers. Simplify your answers.

a) $3 \times \frac{5}{8}$

....

b) $9 \times \frac{3}{4}$

....

c) $8 \times \frac{2}{5}$

....

Finding a fraction of an amount

3. Find these fractions of amounts.

a) $\frac{4}{5}$ of £200

£ ....

b) $\frac{2}{7}$ of 49 sweets

.... sweets

c) $\frac{5}{9}$ of 54 cm

.... cm

ORGANISE

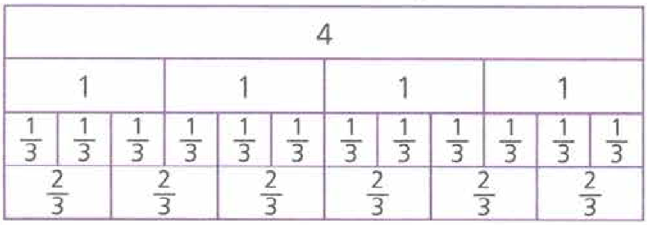
6 Dividing fractions

Dividing an integer by a fraction

Division is another way of asking how many times one quantity fits into another. Dividing an integer by a fraction is asking how many times that fraction fits into the integer.

Work out $4 \div \frac{2}{3}$

To find $4 \div \frac{2}{3}$, a bar with a value of 4 can be divided into parts representing $\frac{2}{3}$



There are 6 sections showing $\frac{2}{3}$

Therefore, $4 \div \frac{2}{3} = 6$

Without a bar model, you can carry out division calculations involving fractions by using the **reciprocal method**. This involves keeping the first number the same, using the reciprocal of the second number, and changing the division to multiplication.

Keep the first number the same
Flip the second number
Change ÷ to ×

To find the reciprocal of a fraction, the numerator becomes the denominator, and the denominator becomes the numerator. Think of the reciprocal of a fraction as the fraction flipped upside down.

Work out $6 \div \frac{2}{3}$

$$6 \div \frac{2}{3} = 6 \times \frac{3}{2}$$

Keep the first value the same; Change ÷ to ×; Flip the second value

$$= \frac{6 \times 3}{2} = \frac{18}{2} = 9$$

Dividing a fraction by a fraction

Dividing a fraction by a fraction means asking how many times one fraction fits into another.

To divide a fraction by a fraction, use the KFC method.

$\frac{3}{4} \div \frac{1}{2}$ means how many halves are in three quarters.

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$$

Keep the first value the same; Change ÷ to ×; Flip the second value

$$= \frac{3 \times 2}{4 \times 1}$$

$$= \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

Work out $\frac{2}{5} \div \frac{7}{9}$

$$\frac{2}{5} \div \frac{7}{9} = \frac{2}{5} \times \frac{9}{7}$$

$$= \frac{2 \times 9}{5 \times 7}$$

$$= \frac{18}{35}$$

You can remember the reciprocal method by KFC: Keep, Flip, Change

Dividing with mixed numbers

If the calculation involves mixed numbers, first convert the mixed number to an improper fraction. Then use the reciprocal method to divide.

Work out $2\frac{1}{3} \div \frac{5}{9}$

$2\frac{1}{3} = \frac{(3 \times 2) + 1}{3} = \frac{7}{3}$ — Convert $2\frac{1}{3}$ to an improper fraction.

$\frac{7}{3} \div \frac{5}{9} = \frac{7}{3} \times \frac{9}{5}$ — Use the reciprocal method.

$= \frac{7 \times 9}{3 \times 5} = \frac{63}{15}$

$\frac{63}{15} = 4\frac{3}{15} = 4\frac{1}{5}$ — Simplify the answer if possible.

RETRIEVE

6 Dividing fractions

Dividing an integer by a fraction

1 Divide these integers and fractions.

a) $9 \div \frac{2}{3}$

....

b) $6 \div \frac{5}{6}$

....

Dividing a fraction by a fraction

2 Divide these fractions.

a) $\frac{1}{6} \div \frac{3}{8}$

....

b) $\frac{1}{9} \div \frac{9}{4}$

....

c) $\frac{1}{2} \div \frac{2}{3}$

....

Dividing with mixed numbers

3 Divide these fractions and mixed numbers.

a) $3\frac{2}{7} \div \frac{1}{2}$

....

b) $5\frac{2}{3} \div \frac{1}{3}$

....

ORGANISE

6 Solving problems with fractions

Comparing and ordering

To compare fractions, remember that they must first have a common denominator. To order a mixture of fractions and decimals, convert them to the same form – either all decimals or all fractions.

Read the question carefully to see if you need to order from greatest to least or from least to greatest.

Five children did the long jump. Their results are below. Order them from first to fifth place.

Abdul: 1.87m Bethany: $1\frac{5}{8}$ m Carli: $\frac{7}{4}$m Dante: 1.7m Ethan: 1.59m

First convert the fractions to decimals:

$\frac{5}{8} = 5 \div 8 = 0.625$, so $1\frac{5}{8} = 1.625$

$\frac{7}{4} = 7 \div 4 = 1.75$

Now write all the values to the same number of decimal places. The most decimal places in the problem is 3, so write all the numbers to 3 d.p.

1.87 = 1.870 1.7 = 1.700 1.59 = 1.590

Order the numbers greatest to smallest. It may help to put the numbers in a place value chart.

Compare the numbers by looking at the digits in the place values from left to right.

1.870, 1.750, 1.700, 1.625, 1.590

The question asked about the children, not the distances, so write the order with their names: Abdul, Carli, Dante, Bethany, Ethan

Fractions of amounts

You will encounter problems in maths and in life where you need to find a fraction of an amount.

If a question mentions a fraction **of** an amount, that's multiplication!

Aisha wants to make 18 cupcakes. How much butter should she use?

Recipe for 24 cupcakes

220 g butter	1 tsp vanilla
220 g sugar	220 g self-raising flour
4 eggs	

First, work out what fraction of the recipe Aisha wants to make. She wants to make 18 out of the 24 cupcakes, so she wants $\frac{18}{24} = \frac{3}{4}$ of the recipe.

Now multiply the amount of butter by $\frac{3}{4}$:

$220\text{g} \times \frac{3}{4} = 165\text{g}$ of butter needed

What fraction is used or remains

You also need to recognise when a problem requires the addition or subtraction of fractions. A problem may ask what fraction remains, in which case you must subtract.

If a question asks how much is used or left, that's addition or subtraction!

Dev makes a jug containing 1.75 L of squash. He and three friends each drink $\frac{2}{5}$ of a litre. How much squash remains?

First convert 1.75 to a fraction: $1.75 = 1\frac{75}{100} = 1\frac{3}{4}$

4 people each drink $\frac{2}{5}$ of a litre so they have drunk $4 \times \frac{2}{5} = \frac{8}{5} = 1\frac{3}{5}$ of a litre

Now subtract the amount they have drunk from the amount originally in the jug: $1\frac{3}{4} - 1\frac{3}{5}$

Convert the mixed numbers to improper fractions with common denominators so that they can be subtracted:

$1\frac{3}{4} = \frac{7}{4} = \frac{35}{20}$ and $1\frac{3}{5} = \frac{8}{5} = \frac{32}{20}$

$1\frac{3}{4} - 1\frac{3}{5} = \frac{35}{20} - \frac{32}{20}$

$= \frac{3}{20}$ of a litre of the squash is left

RETRIEVE

6 Solving problems with fractions

Comparing and ordering

1 Janice needs $\frac{5}{8}$ of a metre of ribbon to complete a project.

She has the following colours and lengths:

Red 0.67 m   Gold $\frac{4}{10}$ m   Blue $\frac{12}{15}$ m   Silver 0.85 m

Which colours could Janice use?

Fractions of amounts

2 A doctor is prescribing medication for a child. She needs to prescribe $\frac{2}{5}$ of a millilitre for every kilogram the child weighs.

The child weighs 30 kg.

How many millilitres should the doctor prescribe?

What fraction is used or remains

3 Siobhan orders two pizzas to share with her friends.

Connor eats $\frac{3}{8}$ of a pizza. Saoirse eats $\frac{5}{12}$ of a pizza. Siobhan eats $\frac{1}{2}$ of a pizza.

How much of a pizza is left? Give your answer as a fraction.

ORGANISE 6

Comparing and ordering numbers in standard form

Comparing when n is positive

Comparing numbers in standard form is similar to comparing decimals:

- Compare the power of 10 (n).
- The greater number will have the greater power of 10.
- If numbers have the same power of 10, compare the first digit (the highest place value), then the second, and so on.

For example:

2×10^5 is greater than 2×10^3 because 5 is greater than 3.

2.3×10^8 is greater than 8.4×10^7 because 8 is greater than 7.

Which is greater, 5.86×10^6 or 5.79×10^6?

Both numbers have a power of 6. — *Look at the powers.*

Both numbers have 5 in the ones place. — *Look at the highest place value.*

5.86×10^6 has an 8 in the tenths place and 5.79×10^6 has a 7 in the tenths place. — *Look at the next place value.*

So, using inequality notation:

$5.86 \times 10^6 > 5.79 \times 10^6$

Comparing when n is negative

Comparing numbers in standard form when n is negative is the same as comparing when n is positive. Remember that the more negative a number is, the smaller it is.

For example:

3×10^{-2} is greater than 3×10^{-4} because −2 is greater than −4.

6.4×10^{-5} is less than 6.4×10^{-3} because −5 is less than −3.

3.8×10^{-8} is greater than 8.3×10^{-12} because −8 is greater than −12.

7×10^{-3} is less than 7×10^2 because −3 is less than 2.

3.46×10^2 is greater than 3.46×10^{-1} because 2 is greater than −1.

Which is greater, 2.33×10^{-5} or 4.33×10^{-5}?

Both numbers have a power of −5.

Comparing the ones places, 2 is less than 4.

So, using inequality notation:

$2.33 \times 10^{-5} < 4.33 \times 10^{-5}$

Ordering numbers in standard form

To order numbers in standard form, identify the greatest and least numbers first, then order from there using the same procedure for comparing numbers in standard form.

Order these numbers from smallest to greatest:

3.1×10^7 9.2×10^7 5.85×10^{-5} 5.42×10^{-4} 3.14×10^{10}

Identify the smallest and greatest numbers in the list.

5.85×10^{-5} is the smallest number because −5 is the smallest power.
3.14×10^{10} is the greatest number because 10 is the greatest power.

Then look at the powers of the remaining numbers and order them.

5.42×10^{-4} has a power of −4.
Both 3.1×10^7 and 9.2×10^7 have a power of 7, but 3.1 is less than 9.2, so 3.1×10^7 is less than 9.2×10^7

5.85×10^{-5} 5.42×10^{-4} 3.1×10^7 9.2×10^7 3.14×10^{10}

Smallest ← → Largest

So, using inequality notation: $5.85 \times 10^{-5} < 5.42 \times 10^{-4} < 3.1 \times 10^7 < 9.2 \times 10^7 < 3.14 \times 10^{10}$

RETRIEVE 6

Comparing and ordering numbers in standard form

Comparing when n is positive

1 Compare each pair of numbers in standard form. Write your answer using inequality notation.

a) 1.35×10^8 and 1.38×10^{10}

b) 6.95×10^5 and 6.9×10^5

c) 9×10^2 and 9×10^3

d) 6.239×10^6 and 1.53×10^5

Comparing when n is negative

2 Compare each pair of numbers in standard form. Write your answer using inequality notation.

a) 4.09×10^8 and 4.09×10^{-10}

b) 9.64×10^{-5} and 9.6×10^{-5}

c) 1.8×10^{-5} and 1.8×10^{-2}

d) 4.21×10^{-6} and 2.592×10^2

Ordering numbers in standard form

3 Order this set of numbers from **greatest to least**. Write your answer using inequality notation.

8.06×10^6 $\quad 2.64 \times 10^0$ $\quad 7.8 \times 10^6$ $\quad 2.6 \times 10^{-9}$ $\quad 3.03 \times 10^{-7}$

ORGANISE

6 Introducing standard form

Standard form and powers of 10

Standard form (or **standard index form**) is a way of writing very large or very small ordinary numbers as **powers of 10** so that they are easier to work with.

Recall powers of 10:

$10^0 = 1$

$10^{-1} = \frac{1}{10} = 0.1$ $\quad$ $10^1 = 10$

$10^{-2} = \frac{1}{10^2} = 0.01$ $\quad$ $10^2 = 10 \times 10 = 100$

$10^{-3} = \frac{1}{10^3} = 0.001$ $\quad$ $10^3 = 10 \times 10 \times 10 = 1000$

... and so on.

A number written in standard form is in the form $A \times 10^n$ where $1 \leqslant A < 10$ and n is a positive or negative integer (or zero). The value of A must be equal to or greater than 1 and less than 10.

If the ordinary number is:

- between 0 and 1, n will be negative
- between 1 and 10, n will be zero
- over 10, n will be positive.

A number written to a negative power of 10 in standard form is a decimal number in ordinary form.

Tick the numbers that are correctly written in standard form. Cross the numbers that are not.

a) 9.85×10^4 ✔

The value of A is 9.85 and the value of n is 4.

b) 6.46×10^{-6} ✔

The value of A is 6.46 and the value of n is –6.

c) 37.73×10^2 ✘

The value of A is **not between** 1 and 10.

Converting numbers between ordinary form and standard form

To convert a number **from ordinary form to standard form**, follow these steps:

- Rewrite the ordinary number with the first non-zero digit in the ones place.
- To find n (the power of 10), look at how many places the digits have moved.
- Decide if the value of n is positive, negative or zero.

Write each number in standard form.

a) 7901

Thousands	Hundreds	Tens	Ones	tenths	hundredths	thousandths
7	9	0	1			
			7 •	9	0	1

The digits have moved by three places and 7901 is greater than 10, so 7901 in standard form is 7.901×10^3

b) 0.027

Thousands	Hundreds	Tens	Ones	tenths	hundredths	thousandths
			0 •	0	2	7
			2 •	7		

The digits have moved by two places and 0.027 is between 0 and 1, so 0.027 in standard form is 2.7×10^{-2}

To convert a number **from standard form to ordinary form**, multiply the value of A by the power of 10. Remember to fill in any empty place values with zeros.

Write each number in ordinary form.

a) 5.13×10^4

5.13×10^4 means $5.13 \times 10 \times 10 \times 10 \times 10$, so each digit moves to the left by four places.

Ten Thousands	Thousands	Hundreds	Tens	Ones	tenths	hundredths	thousandths
				5 •	1	3	
5	1	3	0	0			

Move each digit four places left.

Fill in the tens and ones columns with zeros.

5.13×10^4 is 51 300 as an ordinary number.

b) 1.058×10^{-2}

1.058×10^{-2} means $1.058 \div 10^2$, so each digit moves to the right by two places.

Ones	tenths	hundredths	thousandths	ten thousandths	hundred thousandths
1 •	0	5	8		
0 •	0	1	0	5	8

Move each digit two places right.

Fill in the ones and tenths columns with zeros.

1.058×10^{-2} is 0.01058 as an ordinary number.

RETRIEVE

6 Introducing standard form

Standard form and powers of 10

1 Write each power of 10 as an ordinary number.

a) 10^8 ....

b) 10^{-4} ....

c) 10^6 ....

d) 10^{-8} ....

2 Circle the numbers that are written in standard form.

123.4	1.234×10^{-10}	12 340 000	12.34×10^{-2}
12×10^{34}	1.2×10^{34}	1×10^{23}	4×10^0

Converting numbers between ordinary form and standard form

3 Write each ordinary number in standard form.

a) 5326 ....

b) 72 835 000 ....

c) 0.00726 ....

d) 0.209 ....

4 The following numbers are in standard form. Write them as ordinary numbers.

a) 1.12×10^8 ....

b) 6.18×10^{-4} ....

c) 4.19×10^6 ....

d) 2.05×10^{-3} ....

ORGANISE

5 Direct proportion

Direct proportion word problems

Two quantities are in **direct proportion** when they increase or decrease at the same rate.

Sam can type 80 words in 1 minute. How many words can Sam type in 10 minutes?

$\times 10$: 80 words = 1 minute $\times 10$

800 words = 10 minutes

For every minute, Sam types 80 words. To work out how many words Sam will type in 10 minutes, you need 10 lots of 80.

Kelly works for 5 hours and gets paid £40. How much will Kelly be paid for working 6 hours?

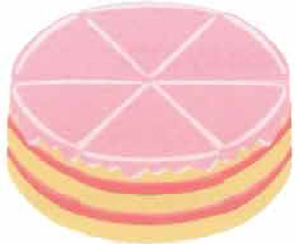
$\div 5$: 5 hours = £40 $\div 5$

$\times 6$: 1 hour = £8 $\times 6$

6 hours = £48

For every hour that Kelly works, she will be paid the same amount. To find out how much Kelly gets paid for 1 hour of work, divide £40 by 5. This means that for every hour Kelly works, she gets paid £8.
To work out how much Kelly will get paid for working 6 hours, you need 6 lots of £8.

400 g of sugar is needed to bake a cake for 8 people. How much sugar is needed to bake a cake for 10 people?



$\div 4$: 400 g = 8 people $\div 4$

$\times 5$: 100 g = 2 people $\times 5$

500 g = 10 people

The **highest common factor** of 8 people and 10 people is 2 people. Therefore, to work out how many grams of sugar are needed for 10 people, find out how many grams of sugar are needed for 2 people, and multiply this by 5.

Direct proportion formula

To express that y is directly proportional to x, the α symbol is used. $y \alpha x$ means that y is directly proportional to x. This can be converted into the formula:

$$y = kx$$

k is a **constant**

A constant is a fixed number that does not change.

A is directly proportional to B and when $A = 30$, $B = 6$. Find the value of A when $B = 10$.

$A \alpha B$

$A = kB$

$30 = k \times 6$

$\div 6 \quad \div 6$

$5 = k$

$A = 5 \times 10$

$A = 50$

To find k, substitute in the values that are given.

S is directly proportional to T and when $S = 10$, $T = 50$. Find the value of S when $T = 60$.

$S \alpha T$

$S = kT$

$10 = k \times 50$

$\div 50 \quad \div 50$

$0.2 = k$

$S = 0.2 \times 60$

$S = 12$

To find k, substitute in the values that are given.

RETRIEVE

5 Direct proportion

Direct proportion word problems

1. If 3 pencils cost 75p, how much do 6 pencils cost?

2. If 4 lollipops cost 80p, how much do 2 lollipops cost?

3. Jeremiah can type 60 words in 1 minute.

   How many words can Jeremiah type in 3 minutes?

4. An electrician works for 3 hours and gets paid £150.

   How much will the electrician be paid for working for 5 hours?

5. If 2 pencils cost 44p, how much will 9 pencils cost?

6. A car travels 120 miles in 4 hours at a steady speed.

   How far does the car travel in 7 hours?

Direct proportion formula

7. A is directly proportional to B and when $A = 24$, $B = 4$.

   Find the value of A when B is 9.

8. X is directly proportional to Y and when $X = 35$, $Y = 7$.

   Find the value of X when Y is 9.

9. P is directly proportional to Q and when $P = 60$, $Q = 120$.

   Find the value of P when Q is 100.

ORGANISE

5 Inverse proportion

Inverse proportion word problems

Two quantities are in **inverse proportion** when as one quantity increases, the other quantity decreases at the **same rate**.

It takes 10 hours for 10 people to decorate some cakes. How long would it take 5 people to decorate the same number of cakes?

÷ 2 (10 people = 10 hours) × 2
5 people = 20 hours

It takes half the number of people double the amount of time (20 hours) to decorate the same number of cakes.

There is enough food to feed 11 chickens for 4 days. How many days of food are there if there are 2 chickens?

÷ 11 (11 chickens = 4 days) × 11
× 2 (1 chicken = 44 days) ÷ 2
2 chickens = 22 days

The fewer chickens there are, the greater the number of days' worth of food there is. There are 22 days of food for 2 chickens.

It takes 12 hours for 5 people to paint a room. How many people would be needed if the room had to be painted in 10 hours?

÷ 6 (12 hours = 5 people) × 6
× 5 (2 hours = 30 people) ÷ 5
10 hours = 6 people

The **highest common factor** of 12 hours and 10 hours is 2 hours. Therefore, to work out how many people will be needed for 10 hours, find out how many people are needed for 2 hours. Then work out how many people are needed for 10 hours.

6 people are needed to complete the job in 10 hours.

Inverse proportion formula

To express that y is inversely proportional to x, the α symbol is used. $y \alpha \frac{1}{x}$ means that y is inversely proportional to x. This can be converted into the formula:

$$y = \frac{k}{x}$$

k is a **constant**

A constant is a fixed number that does not change.

A is inversely proportional to B and when $A = 3$, $B = 9$. Find the value of A when $B = 3$.

$A \alpha \frac{1}{B}$

$A = \frac{k}{B}$

$3 = \frac{k}{9}$

× 9 × 9

$27 = k$

$A = \frac{27}{3}$

$A = 9$

To find k, substitute in the values that are given.

P is inversely proportional to Q and when $P = 3$, $Q = 15$. Find the value of Q when $P = 5$.

$P \alpha \frac{1}{Q}$

$P = \frac{k}{Q}$

$3 = \frac{k}{15}$

× 15 × 15

$45 = k$

$5 = \frac{45}{Q}$

× Q × Q

$5Q = 45$

÷ 5 ÷ 5

$Q = 9$

To find k, substitute in the values that are given.

RETRIEVE

5 Inverse proportion

Inverse proportion word problems

1. It takes 10 people 20 hours to decorate some cakes.

   How long would it take 5 people to decorate the same number of cakes?

2. It takes 4 builders 6 hours to build a wall.

   How long would it take 12 builders to build the same wall?

3. It takes 4 teachers 8 hours to mark some exam papers.

   How long would it take 1 teacher?

4. 3 friends go camping and pack enough water to last for 6 days.

   How long will the same amount of water last if 2 friends go camping?

5. It takes 2 people 5 hours to decorate some cakes.

   How long would it take 5 people to decorate the same number of cakes?

6. 4 people can paint a fence in 3 hours.

   How long will it take 6 people to paint the fence?

Inverse proportion formula

7. A is inversely proportional to B and when $A = 4$, $B = 2$.

   Find the value of A when B is 8.

8. X is inversely proportional to Y and when $X = 3$, $Y = 4$.

   Find the value of X when Y is 2.

9. P is inversely proportional to Q and when $P = 4$, $Q = 5$.

   Find the value of Q when P is 2.

ORGANISE

7 Area and circumference of a circle

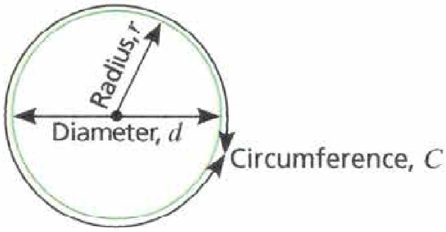
Circumference of a circle

Circumference of a circle = perimeter of circle

Diameter = 2 × radius

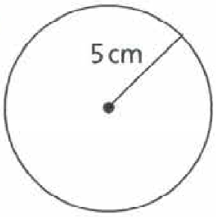
$d = 2r$

Circumference of a circle: $C = 2\pi r$ or $C = \pi d$



Work out the circumference of each circle. Give your answers to 1 decimal place.

a)



You know the radius, so use $C = 2\pi r$

$C = 2\pi r$

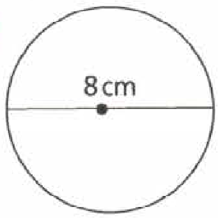
$= 2 \times \pi \times 5$

Use the π key on your calculator.

$= 31.41592\ldots$

$C = 31.4$ cm (1 d.p.)

b)



You know the diameter, so use $C = \pi d$

$C = \pi d$

$= \pi \times 8$

$= 25.13274\ldots$

$C = 25.1$ cm (1 d.p.)

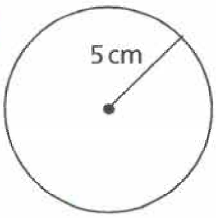
Area of a circle

The area of a circle is the space inside the circumference.

Area of a circle: $A = \pi r^2$

Work out the area of each circle. Give your answers to 1 decimal place.

a)



$A = \pi r^2$

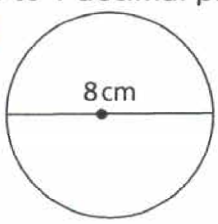
$= \pi \times 5^2$

Use the π key on your calculator.

$= 78.5398\ldots$

$A = 78.5\ \text{cm}^2$ (1 d.p.)

b)



You know the diameter, so find the radius first.

$r = 8 \div 2 = 4$

$A = \pi r^2$

$= \pi \times 4^2$

$= 50.2654\ldots$

$A = 50.3\ \text{cm}^2$ (1 d.p.)

Area and circumference problems

Remember which formula is which.

Area

^{2} in formula and units

$A = \pi r^2$ measured in cm^2 (or km^2 or m^2 or mm^2)

Circumference

No ^{2} in formula or units

$C = 2\pi r$ or $C = \pi d$ measured in cm (or km or m or mm)

A circle has circumference 36 cm. Work out its area. Give your answer to 1 decimal place.

$2\pi r = 36$ Use $C = 2\pi r$ to write an equation.

$\div 2\pi$ $2\pi r = 36$ $\div 2\pi$

$r = \frac{36}{2\pi}$ Solve the equation to find r.

$r = 5.7295\ldots$ cm Use a calculator. Keep several decimal places in your value for r.

$A = \pi r^2$

$= \pi \times 5.7295^2$ Use your value of r.

$= 103.1295\ldots$

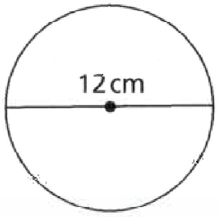
$A = 103.1\ \text{cm}^2$ (1 d.p.) Round to 1 decimal place.

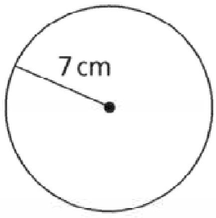
RETRIEVE

7 Area and circumference of a circle

Circumference of a circle

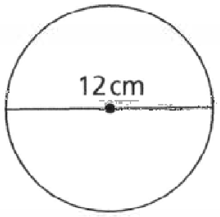
1. Work out the circumference of each circle. Give your answers to 1 decimal place.

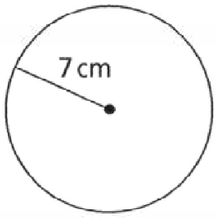
a) 

b) 

Area of a circle

2. Work out the area of each circle. Give your answers to 1 decimal place.

a) 

b) 

Area and circumference problems

3. A circle has circumference 50 cm.

Work out its radius. Give your answer to 1 decimal place.

4. A circle has circumference 72 cm.

Work out its diameter. Give your answer to 1 decimal place.

5. A circle has area 48 cm^2.

Giving your answers to 1 decimal place, work out:

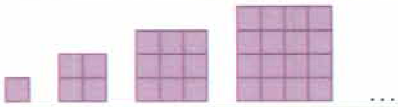
a) its radius **b)** its diameter **c)** its circumference

Key facts and vocabulary

Number

Sequence of square numbers

1, 4, 9, 16, 25, 36, 49, …



Sequence of triangular numbers

1, 3, 6, 10, …

+2 +3 +4 +5

Standard form

A number written as $A \times 10^n$

$1 \leqslant A < 10$ — an integer (n)

Large numbers have positive n: $4.3 \times 10^6 = 4300000$ (6 places)

Numbers less than 1 have negative n: $2.5 \times 10^{-4} = 0.00025$ (4 places)

Algebra

Changing the subject of a formula

Rearranging the terms in a formula so a different variable is on its own on one side of the equals sign

For example: $S = \frac{D}{T}$ (S is the subject) $D = S \times T$ (D is the subject)

Think of rearranging formulae as the same as solving equations. Get the new subject of the formula on its own by using inverse operations.

Compare lines from their equations

Rearrange the equations so both are in the form $y = mx + c$

Compare the gradients (m values)

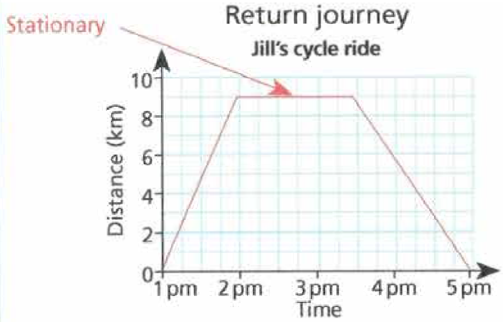
Compare the y-intercepts (c values)

Parallel lines have the same gradient (m value)

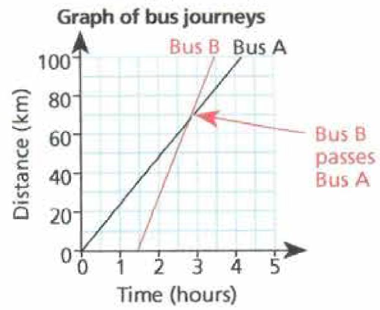
Distance–time graph

Graph showing time on horizontal axis and distance on vertical axis

Return journey



One-way journey



Gradient = speed

Expand a single pair of brackets

Multiply every term inside the bracket by the term outside the bracket

$3(x + 4) = 3x + 12$ $5(2y - 1) = 10y - 5$

Expand two pairs of brackets

Multiply every term in the first bracket by every term in the second bracket

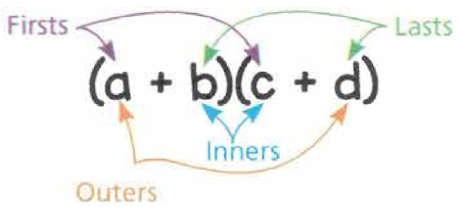
Grid method

$(2x + 3)(x - 4)$

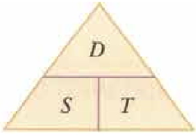
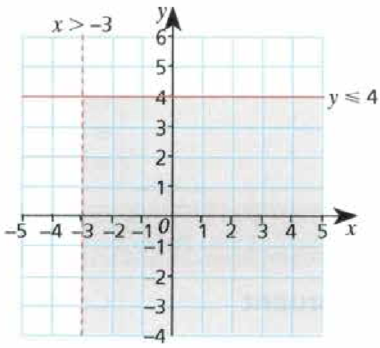
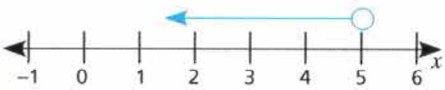
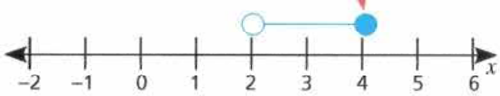
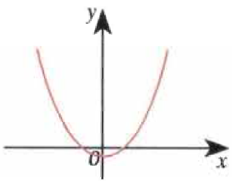
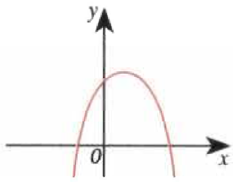
	$2x$	$+3$
x	$2x^2$	$3x$
-4	$-8x$	-12

$= 2x^2 - 5x - 12$

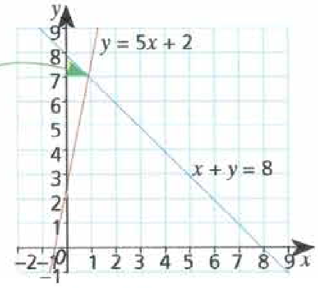
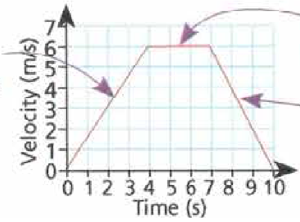
FOIL method



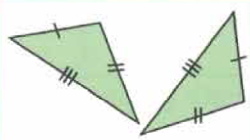
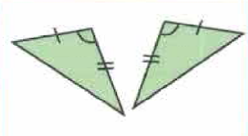
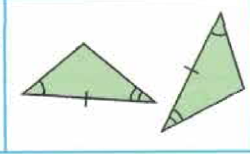
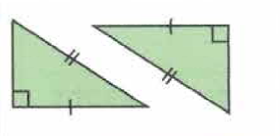
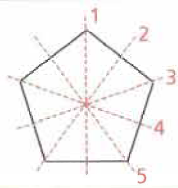
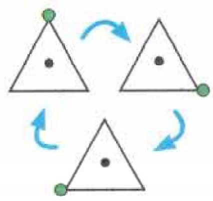
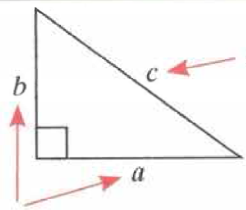
Key facts and vocabulary

Expand three pairs of brackets	Expand the first pair of brackets: $(2x + 3)(x - 4)(x - 3) = (2x^2 - 5x - 12)(x - 3)$ Then multiply every term in the first bracket by every term in the second bracket: $(2x^2 - 5x - 12)(x - 3) = 2x^3 - 11x^2 + 3x + 36$
Formula	A rule connecting two or more variables or quantities The formula for speed is: Speed = $\frac{\text{distance}}{\text{time}}$ 
Inequalities on a graph	For $x > -3$, draw the graph of $x = -3$ with a dashed line; shade the region where $x > -3$ For $y \leqslant 4$, draw the graph of $y = 4$ with a solid line; shade the region where $y \leqslant 4$ For $y \geqslant mx + c$, draw the graph of $y = mx + c$ with a solid line; test points either side of the line to see which side to shade 
Inequalities on a number line	$x < 5$: 5 is not included  $2 < x \leqslant 4$: 4 is included 
Quadratic graphs	Quadratic graph with a positive x^2 term  Quadratic graph with a negative x^2 term  Quadratic graphs are symmetrical U-shaped curves
Solve	To solve an equation, find the value of the unknown letter When solving an equation, carry out the same operations to both sides at each step.
Solve an equation with brackets	Expand the brackets first
Solve an equation with fractions	To 'undo' the fraction, multiply both sides by the denominator When there are fractions on both sides, multiply both sides by the lowest common multiple (LCM) of their denominators
Solve an equation with x on both sides	Use inverse operations to get all the x terms on one side, and all the numbers on the other
Solve an inequality	Solve in the same way as an equation, by doing the same to both sides If you multiply or divide both sides by a negative number, reverse the inequality sign: $-y > 3$ ×−1 ×−1 $y < -3$

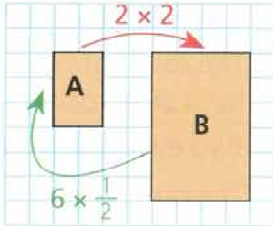
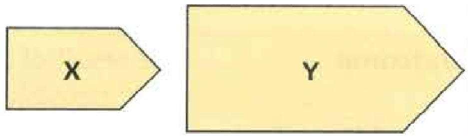
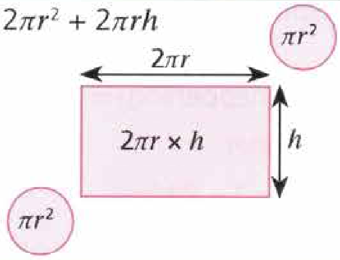
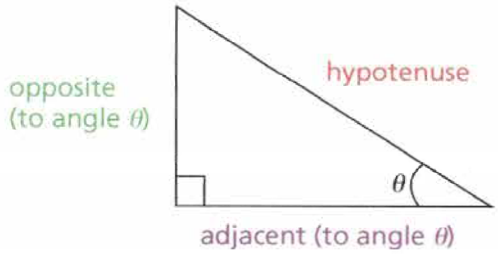
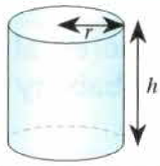
Key facts and vocabulary

Solve simultaneous equations using graphs	Plot the graphs of the two equations The solutions are the x- and y-values where the two lines cross $x = 1$, $y = 7$ 
Velocity–time graph	Graph with time on the horizontal axis and velocity (speed) on the vertical axis A positive gradient (where the line slopes upwards from left to right) means that an object/person is accelerating  A straight line means that an object/person is moving at a constant speed A negative gradient (where the line slopes downwards from left to right) means that an object/person is slowing down (decelerating)

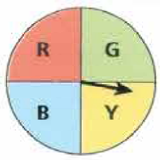
Geometry

Congruent	Congruent shapes are exactly the same shape and size Two triangles are congruent if one or more of these four criteria are true: **Side, Side, Side (SSS)**  **Side, Angle, Side (SAS)**  **Angle, Side, Angle (ASA)**  **Right angle, Hypotenuse, Side (RHS)** 
Cross section	A cross section is the 2D shape that is made when cutting through a 3D shape 
Line of symmetry	Mirror line that divides a shape in half, so each half is a reflection of the other In a regular polygon, number of lines of symmetry = number of sides 
Order of rotational symmetry	The number of times a shape 'lands on itself' when it is rotated a full turn In a regular polygon, the order of rotational symmetry = number of sides 
Pythagoras' theorem	$c^2 = a^2 + b^2$  c is the length of the hypotenuse a and b are the lengths of the two shorter sides

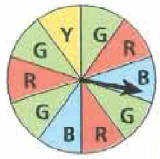
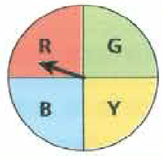
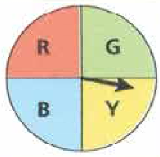
Key facts and vocabulary

Scale factor	The number all side lengths are multiplied by in an enlargement A to B: enlargement scale factor 2 B to A: enlargement scale factor $\frac{1}{2}$ 
Similar	Similar shapes are enlargements of each other • Same angles • Same shape but different sizes • Each side length has been multiplied by the same scale factor 
Surface area	The total area of all the faces of a 3D shape
Surface area of a cylinder	Surface area of a cylinder = $2\pi r^2 + 2\pi rh$ 
Trigonometry ratios	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{opp}}{\text{adj}}$  You can remember these using S O H: $\sin = \frac{\text{opp}}{\text{hyp}}$ C A H: $\cos = \frac{\text{adj}}{\text{hyp}}$ T O A: $\tan = \frac{\text{opp}}{\text{adj}}$
Volume of a cylinder	Volume of a cylinder = area of circular cross section × height $= \pi r^2 h$ 
Volume of a prism	Volume of a prism = area of cross section × length

Probability

Biased	The outcomes are not equally likely; the opposite of 'fair'
Equally likely	All the outcomes have an equal probability of happening In this spinner, the events R, G, B and Y are all equally likely 
Expected result	Expected result = P(event) × number of trials For rolling a dice 30 times, expected number of 3s = $\frac{1}{6} \times 30 = 5$

Key facts and vocabulary

Fair	Unbiased, e.g. a fair coin is equally likely to land on Heads or Tails
Independent events	The result of one event does not change the probability of the second event, e.g. for a dice, rolling a 6 does not change the probability of rolling a 6 next time
Mutually exclusive	Two events that cannot occur at the same time The probabilities of all the mutually exclusive events in a trial add up to 1 For this spinner P(Y) + P(G) + P(R) + P(B) = 1 
Outcome	The result of a probability experiment or trial, e.g. for the experiment 'rolling a dice', possible outcomes are 1, 2, 3, 4, 5 or 6
Probability of an event	$\text{P(event)} = \frac{\text{number of ways the outcome can occur}}{\text{total number of possible outcomes}}$ For this spinner, $\text{P(R)} = \frac{1}{4}$ 
Probability of an event not happening	P(event not happening) = 1 – P(event happening) For this spinner P(not blue) = 1 – P(blue) 
Probability scale	Impossible — Very unlikely — Unlikely — Even chance — Likely — Very likely — Certain 0 (0%) — $\frac{1}{2}$ (50%) — 1 (100%)
Relative frequency	$\text{Relative frequency} = \frac{\text{number of times event occurred}}{\text{total number of trials}}$ The greater the number of trials, the closer the relative frequency is to the theoretical probability
Sample space	Set of all possible outcomes, e.g. for the experiment 'rolling a dice', the sample space is 1, 2, 3, 4, 5, 6 A sample space for two combined events can be shown in a two-way table
Theoretical probability	Probability you calculate using this formula: $\text{P(event)} = \frac{\text{number of ways the outcome can occur}}{\text{total number of possible outcomes}}$
Venn diagram	A diagram showing the relationship between two or more things; this Venn diagram shows whether students had cereal, eggs or neither for breakfast