

Addition and subtracting positive and negative integers

Addition is the process of combining two or more values.

Subtraction is finding the difference between values.

Addition is **commutative**, so $a + b = b + a$.
For example, $2 + 3 = 5$ and $3 + 2 = 5$

A positive number is greater than zero. A negative number is less than zero.

Adding a positive number increases a value:

$$8 + 2 = 10 \qquad 8 + 1 = 9$$

Adding zero does not change a number's value:

$$8 + 0 = 8$$

Adding a negative number decreases a value:

$$8 + -1 = 7 \qquad 8 + -2 = 6$$

Subtracting a positive number decreases a value:

$$9 - 2 = 7 \qquad 9 - 1 = 8$$

Subtracting zero does not change a number's value:

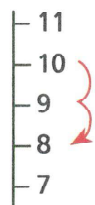
$$9 - 0 = 9$$

Subtracting a negative number increases a value:

$$9 - -1 = 10 \qquad 9 - -2 = 11$$

Work out:

a) $10 + -2$



The value of 10 decreases by 2 so the result is closer to zero.

$$10 + -2 = 8$$

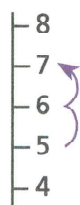
b) $-10 + -2$



The value of -10 decreases by 2 so the result is further away from zero.

$$-10 + -2 = -12$$

c) $5 - -2$



$$5 - -2 = 7$$

d) $-5 - -2$



$$-5 - -2 = -3$$

Addition and subtracting decimals

When adding or subtracting decimal numbers using the column method, align the decimals.

The decimal points must be aligned before adding or subtracting. Use 0 as a placeholder value to make sure that the numbers that are in line have the same place value.

Calculate:

a) $2.043 + 7.238$

$$\begin{array}{r} 2.043 \\ + 7.238 \\ \hline 9.281 \\ 1 \end{array}$$

b) $9.24 - 2.32$

$$\begin{array}{r} 9.24 \\ - 2.32 \\ \hline 6.92 \end{array}$$

c) $1.6 + 3.94$

$$\begin{array}{r} 1.60 \\ + 3.94 \\ \hline 5.54 \\ 1 \end{array}$$

Use a zero as a placeholder.

d) $6.5 - 4.21$

$$\begin{array}{r} 6.50 \\ - 4.21 \\ \hline 2.29 \end{array}$$

e) $76 + 9.17$

$$\begin{array}{r} 76.00 \\ + 9.17 \\ \hline 85.17 \\ 1 \end{array}$$

f) $23 - 2.31$

$$\begin{array}{r} 23.00 \\ - 2.31 \\ \hline 20.69 \end{array}$$

Adding and subtracting positive and negative integers

1 Work out:

a) $4 + -1$

b) $13 + -5$

c) $25 + -11$

d) $1 + -17$

2 Work out:

a) $-9 + -6$

b) $-15 + -4$

c) $-38 + -13$

d) $-4 + -20$

3 Work out:

a) $15 - -2$

b) $61 - -12$

c) $23 - -23$

d) $45 - -15$

4 Work out:

a) $-12 - -8$

b) $-7 - -22$

c) $-3 - -31$

d) $-19 - -1$

Adding and subtracting decimals

5 Work out using the column method:

a) $6.46 + 4.81$

b) $7.8 + 3.2$

c) $7.098 + 5.472$

d) $0.602 + 1.086$

e) $7.4 + 5.09$

f) $9.452 + 8.6$

g) $5 + 0.81$

h) $20 + 5.314$

6 Work out using the column method:

a) $8.25 - 6.13$

b) $7.479 - 3.264$

c) $6.4 - 2.9$

d) $2.32 - 1.93$

e) $5.6 - 2.11$

f) $3.243 - 1.4$

g) $13 - 5.91$

h) $7 - 4.3$

Multiplication and division

Multiplying and dividing with positive and negative integers

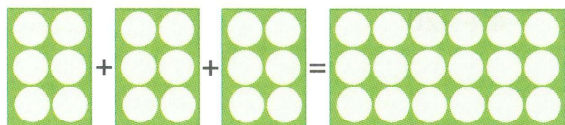
When multiplying or dividing a **positive** number by a **negative** number, the result is **negative**.

When multiplying or dividing a **negative** number by a **negative** number, the result is **positive**.

$$\begin{array}{l} 5 \times 1 = 5 \\ 5 \times 0 = 0 \\ 5 \times -1 = -5 \end{array} \quad \begin{array}{l} -2 \times 1 = -2 \\ -2 \times 0 = 0 \\ -2 \times -1 = 2 \end{array}$$

Multiplication is commutative, so $a \times b = b \times a$.
For example, $4 \times 3 = 12$ and $3 \times 4 = 12$

a) Calculate 6×3



$$6 \times 3 = 18$$

When multiplying a positive number by a positive number, the result is positive ($+ \times + = +$).
 6×3 is the same as adding 3 lots of 6.

b) Calculate -8×3

$$-8 \times 3 = -24 \quad - \times + = -$$

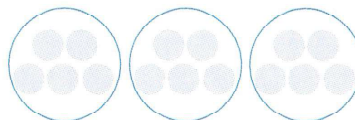
c) Calculate 8×-3

$$8 \times -3 = -24 \quad + \times - = -$$

d) Calculate -7×-4

$$-7 \times -4 = 28 \quad - \times - = +$$

e) Calculate $15 \div 5$



There are 15 dots with 5 dots in each group.
There are 3 groups of dots.

$$15 \div 5 = 3$$

f) Calculate $-20 \div 4$

$$-20 \div 4 = -5 \quad - \div + = -$$

g) Calculate $20 \div -4$

$$20 \div -4 = -5 \quad + \div - = -$$

h) Calculate $-28 \div -7$

$$-28 \div -7 = 4 \quad - \div - = +$$

Multiplying and dividing with decimals

When multiplying a number by another number that is less than 1, the value decreases.

Work out 0.09×0.7

$$\begin{array}{r} 0.09 \times 0.7 \\ \times 100 \quad \times 10 \\ \hline 9 \times 7 = 63 \\ \downarrow \div 1000 \\ = 0.063 \end{array}$$

Multiply both numbers so they are whole numbers.

$100 \times 10 = 1000$, so divide 63 by 1000 to get 0.063

A **divisor** is a number that divides another number and may leave a remainder.

A **dividend** is the number that is being divided.

dividend	divisor	quotient
20	4	= 5

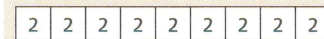
A **quotient** is the result of dividing one number by another.

When dividing with decimals, multiply each number by 10 until the divisor is a whole number.

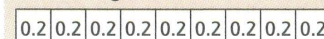
a) Work out $1.8 \div 0.2$

$$\begin{array}{r} 1.8 \div 0.2 \\ \times 10 \quad \times 10 \\ \hline 18 \div 2 = 9 \\ 1.8 \div 0.2 = 9 \end{array}$$

2 goes into 18 nine times.



0.2 also goes into 1.8 nine times.



The calculations are equivalent.

b) Work out $1.36 \div 0.4$

$$\begin{array}{r} 1.36 \div 0.4 \\ \times 10 \quad \times 10 \\ \hline 13.6 \div 4 \end{array}$$

$$\begin{array}{r} 03.4 \\ 4 \overline{) 13.6} \\ \underline{4} \\ 13 \\ \underline{12} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

Ensure that the decimal points are lined up.

$$\text{So, } 1.36 \div 0.4 = 3.4$$

Multiplication and division

Multiplying and dividing with positive and negative integers

1 Work out:

a) 7×8

b) 9×6

c) -6×2

d) -5×10

2 Work out:

a) 7×-2

b) 4×-8

c) -2×-3

d) -12×-8

3 Work out:

a) $12 \div 2$

b) $18 \div 9$

c) $-49 \div 7$

d) $-63 \div 7$

4 Work out:

a) $108 \div -12$

b) $30 \div -6$

c) $-90 \div -9$

d) $-56 \div -7$

Multiplying and dividing with decimals

5 Work out:

a) 8×0.3

.....

b) 0.5×0.7

.....

c) 0.03×0.9

.....

d) 32×0.1

.....

e) 0.9×0.6

.....

f) 0.11×0.4

.....

6 Work out:

a) $8 \div 0.4$

.....

b) $12 \div 0.6$

.....

c) $3.2 \div 0.8$

.....

d) $8.8 \div 0.11$

.....

e) $6.4 \div 0.08$

.....

f) $2.14 \div 0.2$

.....

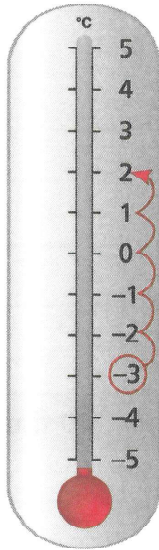
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Negative numbers

Positive and negative temperatures and numbers

Temperatures below freezing (0°C) are negative.

The temperature is -3°C . It rises by 5° . What is the new temperature?



Start at -3°C and count up 5°

The new temperature is 2°C .

Write these temperatures in order, from coldest to hottest:

-5°C 7°C -9°C 0°C 3°C

Find the numbers on a number line.



-9°C -5°C 0°C 3°C 7°C

Write the correct sign ($<$ or $>$) between each pair of numbers:

-4 -8

-10 -2

Find the numbers on a number line.



$-4 > -8$

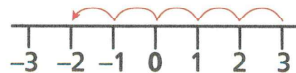
$-10 < -2$

Adding and subtracting negative numbers

Adding a negative number is the same as subtracting, e.g. $3 + -2 = 3 - 2 = 1$

Subtracting a negative number is the same as adding, e.g. $3 - -2 = 3 + 2 = 5$

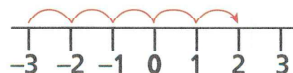
a) Work out $3 + -5$



Replace $+ -$ with $-$

$3 + -5 = 3 - 5 = -2$

b) Work out $-3 + 5$



$-3 + 5 = 2$

c) Work out $3 - -5$

Replace $--$ with $+$

$3 - -5 = 3 + 5 = 8$

Multiplying and dividing negative numbers

positive \times or $+$ negative = negative

e.g. $6 \times -3 = -18$

$6 + -3 = -2$

negative \times or $+$ positive = negative

e.g. $-10 \times 2 = -20$

$-10 + 2 = -5$

negative \times or $+$ negative = positive

e.g. $-10 \times -5 = 50$

$-10 + -5 = 2$

Each counter represents -1 . Write two multiplication facts and two division facts for this array.



$4 \times -2 = -8$

$-8 \div 4 = -2$

$2 \times -4 = -8$

$-8 \div 2 = -4$

Work out:

a) 3×-5 $3 \times -5 = -15$

b) -3×5 $-3 \times 5 = -15$

c) -3×-5 $-3 \times -5 = +15$

d) $12 \div -3$ $12 \div -3 = -4$

e) $-12 \div 3$ $-12 \div 3 = -4$

f) $-12 \div -3$ $-12 \div -3 = +4$

You don't need to write the $+$ sign in your answer.

Negative numbers

Positive and negative temperatures and numbers

1 Use the thermometer to help answer these questions.

a) The temperature is -2°C . It rises by 4° .

What is the new temperature?

b) The temperature is 3°C . It falls by 7° .

What is the new temperature?

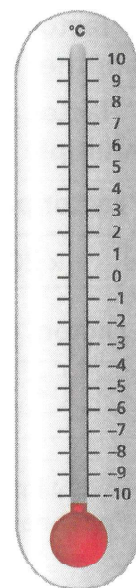
c) What is the difference in temperature between these two cities?

London: 10°C Oslo: -5°C

d) Write these temperatures in order, from hottest to coldest:

-4°C 8°C -7°C 1°C -9°C

.....



2 Write the correct sign (< or >) between each pair of numbers.

a) 5 9

b) -5 -9

c) 6 -27

d) -42 31

Adding and subtracting negative numbers

3 Work out:

a) $5 + -5$

b) $7 - -4$

c) $250 + -100$

d) $-6 + 4$

e) $-13 - -3$

f) $14 + -6 - 2$

Multiplying and dividing negative numbers

4 Work out:

a) $-30 \div 10$

b) -9×2

c) $-100 \div -20$

d) -6×5

e) -5×-7

f) $28 \div -4$

5 Work out:

a) $-4 + 10 \div 5$

b) $-5 \times (3 - 7)$

c) $3 \times -6 - 24 \div -8$

d) $15 \div -5 + 4 \times -3$

6 Fill in the missing numbers in these calculations.

a) $8 + 2 - 2 - \dots = 0$

b) $-5 + 5 - 2 + \dots = 0$

c) $12 + 5 - 12 - \dots = 0$

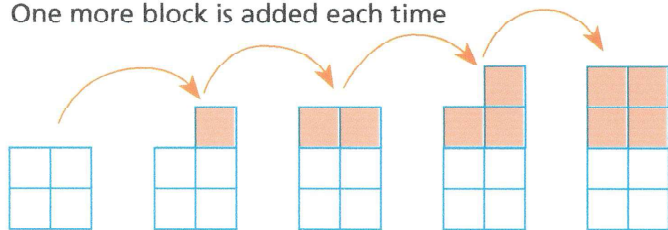
d) $-270 + 70 + \dots = 0$

2 Introducing sequences

What is a sequence?

A **sequence** is a pattern of numbers or images appearing in a special order. The numbers in a sequence are called terms. An ellipsis (...) at the end of the sequence shows that it continues.

One more block is added each time



In an **arithmetic sequence** (also called a **linear sequence**), the terms increase or decrease by the same value (the **common difference**) each time.

1, 3, 5, 7, 9, ... is an arithmetic sequence in which each term increases by 2.

25, 20, 15, 10, 5, ... is an arithmetic sequence in which each term decreases by 5.

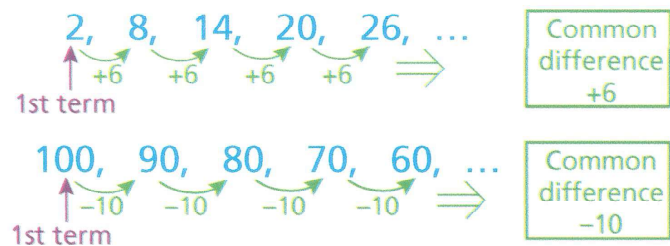
In a **geometric sequence**, each term is multiplied or divided to get the next term. The number the terms are multiplied or divided by is called the **common ratio**.

2, 4, 8, 16, 32, ... is a geometric sequence in which each term is multiplied by 2 to get the next term.

If a sequence is arithmetic, it is increasing or decreasing by a constant value.

Describing and generating arithmetic sequences

An arithmetic sequence can be described in words or by giving the **first term** and the common difference, or **term-to-term rule**.



To **generate a sequence** means to write down the terms. You need the first term and the relationship between the terms (the term-to-term rule).

The first term of a sequence is 15 and the value of each term is 4 less than the previous term. Generate the first five terms.

- 1st term is 15
- 2nd term is $15 - 4 = 11$
- 3rd term is $11 - 4 = 7$
- 4th term is $7 - 4 = 3$
- 5th term is $3 - 4 = -1$
- The sequence is 15, 11, 7, 3, -1, ...

This is an arithmetic sequence as each term is decreasing by the same value.

Describing and generating other sequences

A **geometric sequence** can be described by stating the first term and the number each term is multiplied (or divided) by to get the next term.

The sequence 1, 4, 16, 64, ... starts with 1 and each term is multiplied by 4 to create the next term.

Some sequences are neither arithmetic nor geometric. In the **Fibonacci sequence**, the previous two terms are added together to create the next term. The first eight terms in the sequence are:

1, 1, 2, 3, 5, 8, 13, 21

$1 + 1 = 2, 1 + 2 = 3,$
 $2 + 3 = 5, 3 + 5 = 8,$ etc.

The first term of a sequence is 100 and each term is half the previous term. Write the first five terms.

- 1st term is 100
- 2nd term is $100 \div 2 = 50$
- 3rd term is $50 \div 2 = 25$
- 4th term is $25 \div 2 = 12.5$
- 5th term is $12.5 \div 2 = 6.25$
- The sequence is 100, 50, 25, 12.5, 6.25, ...

This is a geometric sequence as each term is being divided by 2.

Any sequence can be written if you know how it starts and the relationship between the terms.

Introducing sequences

What is a sequence?

- 1 Identify whether the following sequences are **arithmetic** or **geometric**. Circle your answers.
- | | | | |
|----|----------------------|------------|-----------|
| a) | 2, 10, 50, 250, 1250 | Arithmetic | Geometric |
| b) | 6, 1, -4, -9, -14 | Arithmetic | Geometric |
| c) | 2, 4, 6, 8, 10 | Arithmetic | Geometric |
| d) | 16, 8, 4, 2, 1 | Arithmetic | Geometric |

Describing and generating arithmetic sequences

- 2 Describe each sequence by stating the first term and the common difference.

a) 7, 12, 17, 22, 27, ...

.....

b) 11, 7, 3, -1, -5, ...

.....

- 3 Write down the first five terms of each sequence.

a) First term 77, common difference -7

.....

b) First term -6, common difference +3

.....

Describing and generating other sequences

- 4 Describe each geometric sequence by stating the first term and the relationship between the terms.

a) 3, 9, 27, 81, 243, ...

.....

b) 54, 18, 6, 2, $\frac{2}{3}$, ...

.....

- 5 Write down the first five terms of a sequence with first term -1, multiplying each term by 3.
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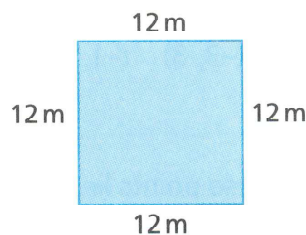
Regular polygons

Perimeter is the total distance around the outside of a **2D shape**. Add the lengths of a shape to find its perimeter.

A **regular polygon** is a 2D shape in which all **sides are the same length** and **interior angles are equal**.

Perimeter is a one-dimensional measure. It is always measured in, for example, cm (not cm² or cm³).

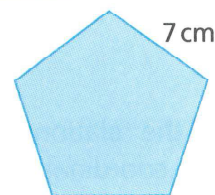
What is the perimeter of this square?



$$\text{Perimeter} = 12\text{ m} + 12\text{ m} + 12\text{ m} + 12\text{ m} = 48\text{ m}$$

Calculate the perimeter of this regular polygon. The length of each side is 7 cm.

$$\text{Perimeter} = 7\text{ cm} + 7\text{ cm} + 7\text{ cm} + 7\text{ cm} + 7\text{ cm} = 35\text{ cm}$$

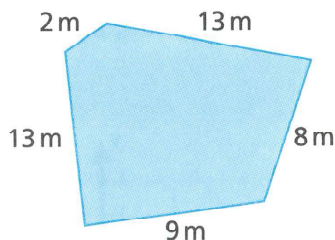


Irregular polygons

An **irregular polygon** is a 2D shape that does **not have all sides equal in length** and all **angles are not equal**.

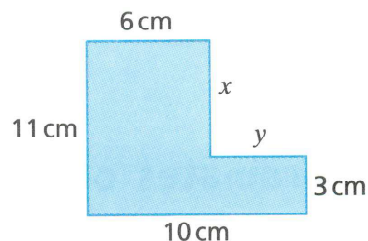
You may need to find unknown lengths in order to calculate the perimeter.

What is the perimeter of this pentagon?



$$\text{Perimeter} = 13\text{ m} + 8\text{ m} + 9\text{ m} + 13\text{ m} + 2\text{ m} = 45\text{ m}$$

What is the perimeter of this shape?



$$\begin{aligned} \text{Side length } x &= 11\text{ cm} - 3\text{ cm} \\ &= 8\text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Side length } y &= 10\text{ cm} - 6\text{ cm} \\ &= 4\text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= \\ &6\text{ cm} + 11\text{ cm} + 10\text{ cm} + 3\text{ cm} + 4\text{ cm} + 8\text{ cm} = 42\text{ cm} \end{aligned}$$

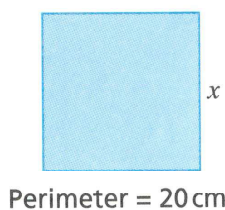
Finding unknown lengths given the perimeter

To find the side length of a regular polygon, divide the perimeter by its number of sides.

Find the side length x of the square given the perimeter is 20 cm.

$$20\text{ cm} \div 4 = 5\text{ cm}$$

$$x = 5\text{ cm}$$



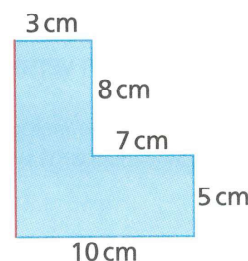
In a regular polygon, you can divide the perimeter by the number of sides because all side lengths are equal.

To find the unknown length of an irregular polygon, add together the lengths of all given sides and subtract from the perimeter.

Find the unknown length of the irregular polygon given the perimeter is 46 cm.

$$\begin{aligned} \text{The given lengths total} \\ 3\text{ cm} + 8\text{ cm} + 7\text{ cm} + 5\text{ cm} \\ + 10\text{ cm} &= 33\text{ cm} \\ 46\text{ cm} - 33\text{ cm} &= 13\text{ cm} \end{aligned}$$

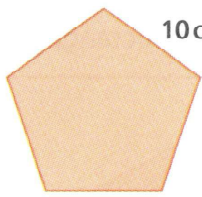
$$\text{Unknown length} = 13\text{ cm}$$



Regular polygons

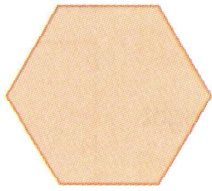
1 Find the perimeter of the following regular polygons.

a)  10 cm

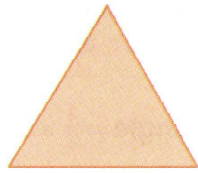


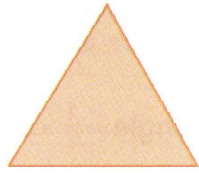
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b)  7 cm



.....

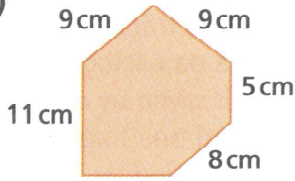
c)  7 mm

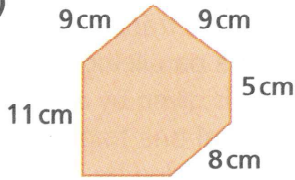


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Irregular polygons

2 Find the perimeter of the following irregular polygons.

a)  9 cm 9 cm 5 cm 8 cm 8 cm 11 cm

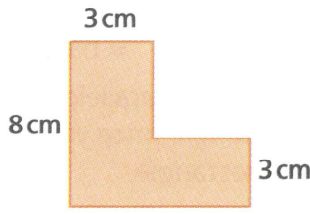


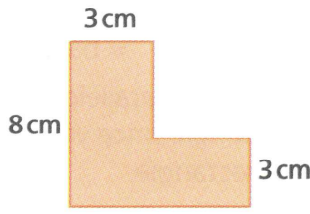
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b)  6 cm 2 cm



.....

c)  3 cm 8 cm 7 cm 3 cm

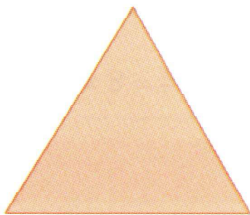


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Finding unknown lengths given the perimeter

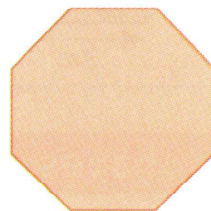
3 Find the length of one side of these regular polygons.

a) Perimeter = 36 cm



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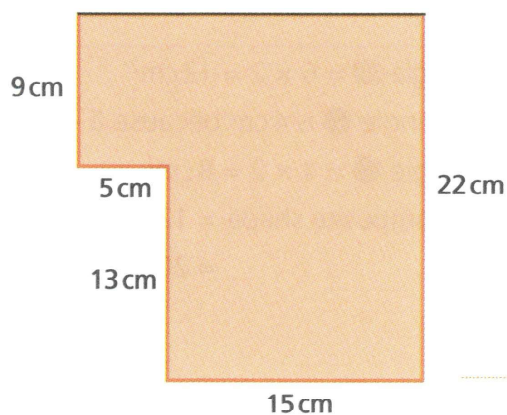
b) Perimeter = 56 m



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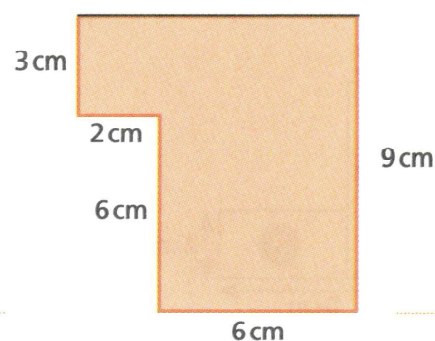
4 Find the lengths of the unmarked sides of these irregular polygons.

a) Perimeter = 84 cm



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b) Perimeter = 34 cm



.....

Area of a square, rectangle and triangle

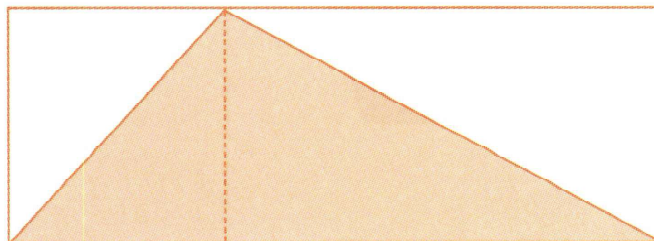
Area of a **square** or a **rectangle** = length \times width

Area is a **two-dimensional** measure, so is measured in, for example, cm^2 , not cm or cm^3

Area of a **triangle** = $\frac{1}{2} \times$ base \times perpendicular height

The **perpendicular height** of a triangle is the height that meets the base at 90° , not the slanted height.

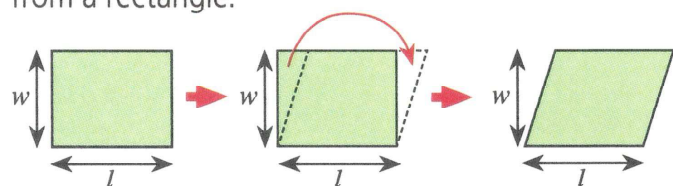
Each part of the triangle is half of the smaller rectangle in which it sits and so the whole triangle is half the area of the large rectangle.



Area of a parallelogram and trapezium

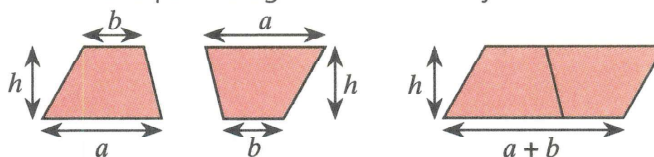
Area of a **parallelogram** = length \times width
 = base \times perpendicular height

The area of a parallelogram is the same as the area of a rectangle since you can form a parallelogram from a rectangle.



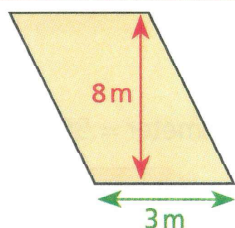
Area of a **trapezium** = $\frac{(a+b)}{2} h$

To derive this formula, join two trapezia to form a parallelogram. The area for a parallelogram is length \times width. The length is given by $a + b$ and the width by h . To find the area of one trapezium, the area of the parallelogram is divided by 2.



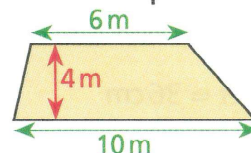
Find the area of the parallelogram.

Area = $3 \times 8 = 24 \text{ m}^2$



Find the area of the trapezium.

Area = $\frac{(a+b)}{2} h = \frac{(6+10)}{2} \times 4 = 8 \times 4 = 32 \text{ m}^2$

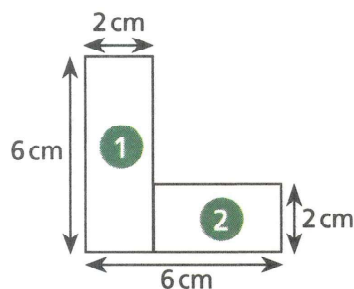


Area of composite shapes

A **composite shape** (or compound shape) is a shape made up of two or more simple shapes.

Split up the composite shape into simple shapes and number them. Then find the area of both shapes and add them together.

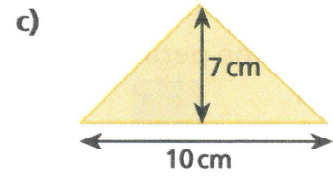
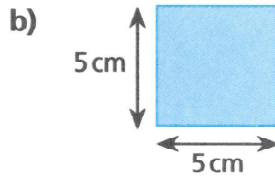
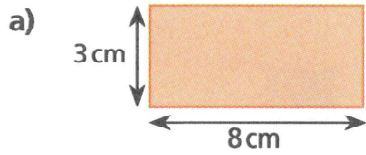
What is the area of this composite shape?



Area of rectangle ① = $6 \times 2 = 12 \text{ cm}^2$
 Length of rectangle ② is 4 cm because $6 - 2 = 4$
 Area of rectangle ② = $4 \times 2 = 8 \text{ cm}^2$
 Total area of composite shape = $12 \text{ cm}^2 + 8 \text{ cm}^2 = 20 \text{ cm}^2$

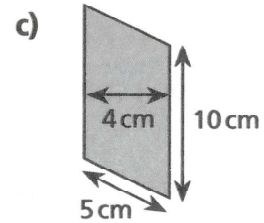
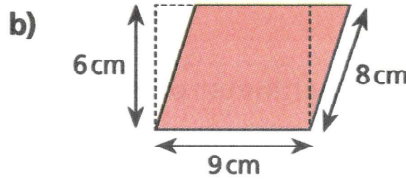
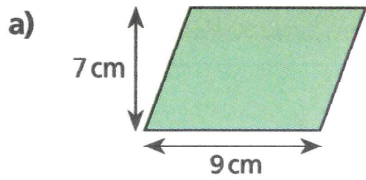
Area of a square, rectangle and triangle

1 Find the areas of the following shapes.

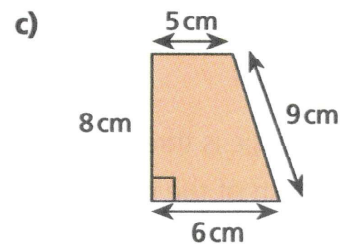
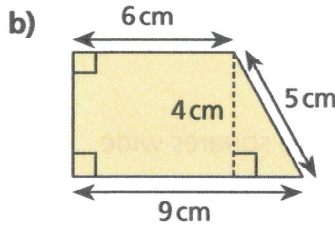
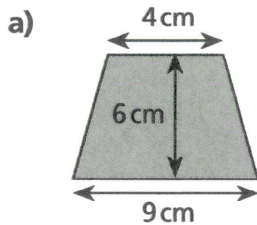


Area of a parallelogram and trapezium

2 Find the areas of the following parallelograms.



3 Find the areas of the following trapeziums.



Area of composite shapes

4 Find the areas of the following composite shapes.

