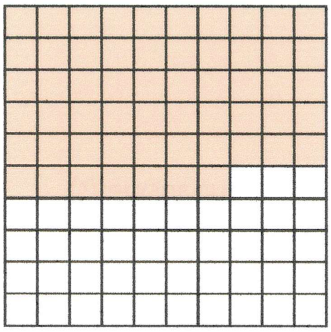
5 Percentages

Fractions, decimals and percentages

Percent means '**out of 100**'. Percentages, fractions and decimals are all ways to represent part of a whole, so percentages can also be written as equivalent fractions and decimals.

57 out of 100 squares (57%) are shaded



Write the amount of cake that has been eaten as a fraction, a decimal and a percentage.

3 out of 8 slices have been eaten. That means $\frac{3}{8}$ of the cake has been eaten.

$\frac{3}{8} = 3 \div 8$ $\quad$ $8\,\overline{)\,3\,.\,{}^{3}0\,{}^{6}0\,{}^{4}0}$ gives 0.375 $\quad$ $\frac{3}{8} = 0.375$

$0.375 \times 100 = 37.5\%$

Conversion	Method
Decimal to percentage	Multiply by 100, e.g. $0.75 \times 100 = 75\%$
Decimal to fraction	Use the smallest decimal place value as the denominator and digits as the numerator, e.g. $0.75 = \frac{75}{100} = \frac{3}{4}$ (the 5 is in the Hundredths place). Divide by 25 to simplify $\frac{75}{100}$
Percentage to decimal	Divide by 100, e.g. $75\% \div 100 = 0.75$
Percentage to fraction	Use 100 as the denominator and the percentage as the numerator, e.g. $75\% = \frac{75}{100}$
Fraction to decimal	Divide the numerator by the denominator, e.g. $\frac{3}{4} = 3 \div 4$ $\quad$ $4\,\overline{)\,3\,.\,{}^{3}0\,{}^{2}0}$ gives 0.75 $\quad$ $\frac{3}{4} = 0.75$
Fraction to percentage	Convert to a decimal then multiply by 100, e.g. $\frac{3}{4} = 0.75$; $0.75 \times 100 = 75\%$

One number as a percentage of another number

To find what percent one number is of another, divide the first number by the second and multiply by 100.

7 out of 10 beads in a bag are red.

This is $\frac{7}{10}$ as a fraction. As a decimal, $\frac{7}{10} = 0.7$

$0.7 \times 100 = 70\%$, so 70% of the beads are red.

To find what percent 18 is of 12, divide $18 \div 12 = 1.5$, then $1.5 \times 100 = 150\%$. 18 is 150% of 12.

A shirt that originally cost £20 is reduced by £3 in a sale. By what percentage has the price decreased?

Find what percent £3 is of £20.

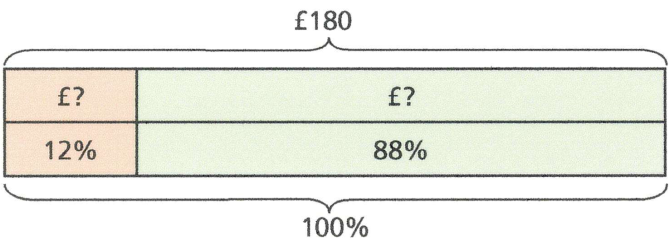
$\frac{3}{20} = 3 \div 20 = 0.15$, then $0.15 \times 100 = 15\%$

The price of the shirt has decreased by 15%.

Finding percentages of a number or an amount

To find a percentage of an amount, convert the percentage to a decimal or fraction and multiply by the amount.

This bar model shows finding 12% of £180.



To find 12% of £180, convert 12% to a decimal or fraction, then multiply by 180. 12% of £180 = £21.60

To find 10% of a number, × by 0.1, or ÷ by 10.

To find 5% of a number, halve 10% of the number.

To find 1% of a number, × by 0.01, or ÷ by 100.

Find 17% of 160.

17% of 160 = 10% of 160 + 5% of 160 + 2% of 160

10% of $160 = 160 \div 10 = 16$

5% of $160 = 16 \div 2 = 8$

2% of $160 = 1.6 \times 2 = 3.2$

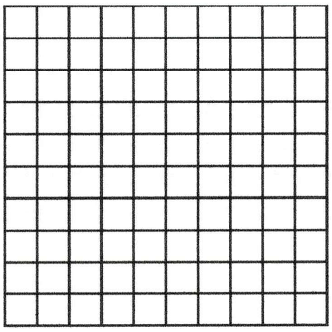
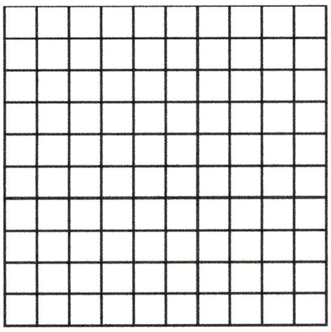
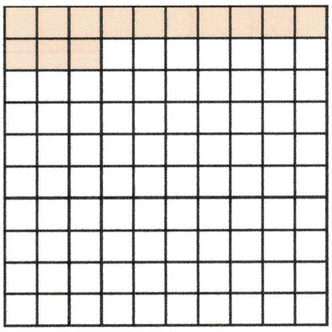
So 17% of $160 = 16 + 8 + 3.2 = 27.2$

RETRIEVE

5 Percentages

Fractions, decimals and percentages

1 Complete the table.

Fraction	$\frac{19}{50}$		
Decimal		0.74	
Percentage			
Diagram			

One number as a percentage of another number

2 Find the percent by which each of these items is discounted. Give your answers to the nearest percent.

a) A shirt is originally £18 and is decreased by £2.

....

b) A pair of trousers is originally £38 and is decreased by £5.

....

Finding percentages of a number or an amount

3 A theme park offers a discount of 8% for tickets bought online.
A family of 1 adult and 3 children plan to go to the theme park.

How much will they save by buying their tickets online?

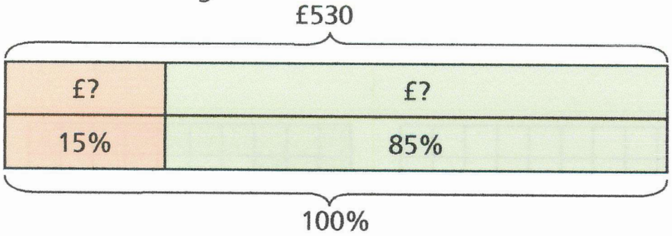
....

5 Percentage changes

Percentage decrease

Prices may be reduced by a certain percentage; this is an example of **percentage decrease**.

This bar model represents a £530 TV decreased in price by 15%. £530 is 100% of the original cost. The price is reduced by 15%, so the sale price is 85% of the original.



To find the new price, either find 15% of the original price and subtract it from the original price, or find 85% of the original price.

To find the new price, use one of these methods:

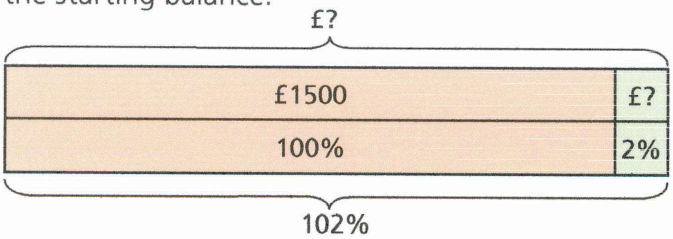
Convert 15% to a decimal: 15% = 15 ÷ 100 = 0.15 Find the amount of the decrease: £530 × 0.15 = £79.50 Find the new price: £530 – £79.50 = £450.50	Find the percentage the sale price is of the original price: 100% – 15% = 85% Convert 85% to a decimal: 85% = 85 ÷ 100 = 0.85 Find the new price: £530 × 0.85 = £450.50

To find the new value after a percentage decrease, multiply the original value by the percent remaining after the decrease (100% – percentage decrease).

Percentage increase

Prices may rise by a certain percentage; this is called **percentage increase**. Interest in a bank account is also an example of percentage increase. The bank pays interest based on a percentage of the balance. When this interest is paid at the end of the year, it is called **simple interest**.

This bar model shows the interest earned on a £1500 bank account. The new balance is 2% more than at the start. So the new balance is 102% of the starting balance.



To find the new balance, use one of these methods:

Convert 2% to a decimal: 2% = 2 ÷ 100 = 0.02 Find the amount of interest earned: £1500 × 0.02 = £30 Add to the original balance to find the new balance: £1500 + £30 = £1530	Find the percentage the new balance is of the original balance: 100% + 2% = 102% Convert 102% to a decimal: 102% = 102 ÷ 100 = 1.02 Find the new balance: £1500 × 1.02 = £1530

To find the new value after a percentage increase, multiply the original value by (100% + the percentage increase).

Finding the percentage change

The percentage by which a value has increased or decreased is the **percentage change**. It is the same as asking what percentage one number is of another.

$$\% \text{ change} = \frac{\text{amount of increase or decrease}}{\text{original value}} \times 100$$

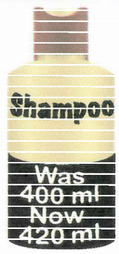
The price of a video game has been reduced by £5. The original price was £40.

$\% \text{ change} = \frac{\text{amount of increase or decrease}}{\text{original value}} \times 100$

$\% \text{ decrease} = \frac{5}{40} \times 100 = 12.5\%$

The price has decreased by 12.5%

Find the percentage increase in the amount of shampoo in the bottle.



Amount of increase:

420 ml – 400 ml = 20 ml

$\% \text{ change} = \frac{\text{amount of increase or decrease}}{\text{original value}} \times 100$

$\% \text{ increase} = \frac{20}{400} \times 100 = 5\%$

The amount of shampoo has increased by 5%

RETRIEVE

5 Percentage changes

Percentage decrease

1. A furniture shop is having a sale. The original prices are shown.

Calculate the new price of each item.

a) 

b) 

Percentage increase

2. A manufacturer is increasing the amounts in its food packages.

a) Increase the amount of cereal by 8%

b) Increase the amount of porridge by 20%

Finding the percentage change

3. An electronics store is having a sale on TVs.

Calculate the percentage change of each of these discounts given the original prices indicated. Give your answers to the nearest percent.

a) A TV originally costs £2500 and is being discounted by £300.

b) A TV originally costs £1700 and is now £1350.

ORGANISE 5

Repeated percentage change and finding the original value

Percentage multipliers

An efficient way of finding a new value after a percentage change is to multiply by the percentage the new value represents; this is called using a **multiplier**.

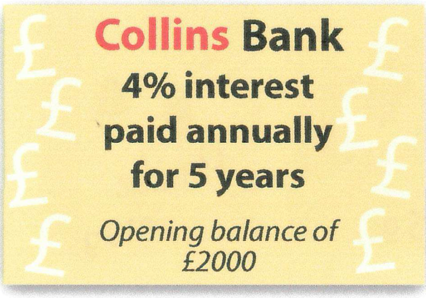
To find the mass after a 20% decrease from 200 g, recognise that a 20% decrease means that the new mass is 80% of the original. So multiply 200 × 0.8 = 160 g. The value 0.8 is the multiplier.

To find the volume after a 10% increase from 300 ml, recognise that a 10% increase means the new volume is 110% of the original. So multiply 300 × 1.1 = 330 ml. The value 1.1 is the multiplier.

Repeated percentage change

Repeated percentage change is when an amount is repeatedly changed by a percentage increase or decrease. For example, a bank may pay interest on a balance year after year, including on the interest earned in previous years; this is called **compound interest**.

£2000 is invested in the bank account advertised below. The bank pays compound interest.



a) How much will be in the account after 3 years?

A 4% increase means the new balance is 104% of the previous balance. The multiplier is 1.04. The value is increased by 4% each year.

New value in year 1:
£2000 × 1.04 = £2080

New value in year 2:
£2000 × 1.04 × 1.04 = £2163.20

New value in year 3:
£2000 × 1.04 × 1.04 × 1.04
= £2249.73

Each year, the previous balance is multiplied by 1.04. A faster way of writing this is £2000 × 1.04^x (where x is the number of years).

b) How much will be in the account after 5 years?

After 5 years,
£2000 × 1.04^5 = £2433.31

To quickly find a repeated percentage change:
New value = original value × multiplier^x (where x is the number of changes)

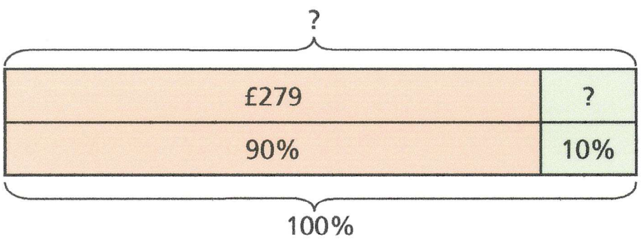
Finding the original value

Finding the original value of a percentage change involves using **reverse percentages**. Think about the calculation required to find the new value and use inverse operations to find the original value.

To find the original value, use the multiplier and inverse operations.

A games console is in a 10% off sale. Its new price is £279. Calculate the original price.

To find the multiplier, think about the percentage that the new price represents of the original price.



The multiplier is 0.9

Say the original value is x. To find the sale price, the calculation would be x × 0.9 = £279

x × 0.9 = £279
÷ 0.9 ÷ 0.9
x = £310

Use inverse operations: divide both sides by 0.9

The original price was £310.

RETRIEVE 5

Repeated percentage change and finding the original value

Percentage multipliers

1. Write down the multiplier for each of these percentage changes.

   a) Increase by 56%

   b) Increase by 32%

   c) Decrease by 4%

   d) Decrease by 21%

Repeated percentage change

2. A bank account pays compound interest of 3% a year.

   If no additional deposits are made, find the balance after 5 years with a starting balance of £1200.

3. A car depreciates (goes down) in value by 8% each year.
   The car costs £20 000 when new.

   Calculate the value of the car after 10 years.

Finding the original value

4. A shop is having a sale of 10% off all mobile phones.
   A mobile phone costs £180 in the sale.

   What was the original price?

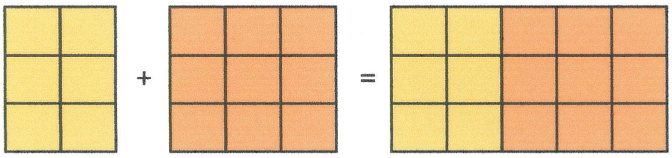
5. A computer store is increasing its prices by 8% on all items.
   After the increase, a laptop costs £850.

   What was the price before the increase?

4 Solving equations with brackets

Expanding brackets using the distributive law

The **distributive law** says that the product of two numbers multiplied together is equal to the product of those numbers split into groups (**partitioned**).



$3 \times 2 \quad + \quad 3 \times 3 \quad = \quad 3 \times 2 + 3 \times 3 = 15$

$3 \times (2 + 3) = 15$

The distributive law can be shown without visual aids by **expanding the bracket**.

$$3(2 + 3) = (3 \times 2) + (3 \times 3)$$
$$= 6 + 9$$
$$= 15$$

The same rule applies when there are variables inside or outside the brackets. $5(2x + 3)$ can be expanded using an area model. Draw a large rectangle partitioned into smaller rectangles and find the area of each.

	$2x$	3
5	$10x$	15
	$5 \times 2x$	5×3

$$5(2x + 3)$$
$$= (5 \times 2x) + (5 \times 3)$$
$$= 10x + 15$$

To expand a pair of brackets with more than two terms, you must multiply by each term.

To expand $4(2x + 3y - 8)$, multiply 4 by each term in the bracket.

	$2x$	$3y$	-8
4	$8x$	$12y$	-32
	$4 \times 2x$	$4 \times 3y$	4×-8

$$4(2x + 3y - 8)$$
$$= 8x + 12y - 32$$

Multiply what is outside the brackets by everything inside the brackets.

When expanding brackets, remember that:

- multiplying or dividing two negative numbers gives a positive number
- multiplying or dividing a negative and a positive gives a negative number
- adding a negative number is the same as subtracting
- subtracting a negative number is the same as adding.

+ × + = +
− × − = +
− × + = −
+ × − = −

Expand $3m(2 - 7n)$

Multiply $3m$ by each term inside the bracket and carry the negative sign through to the answer.

$$3m(2 - 7n) = (3m \times 2) + (3m \times -7n)$$
$$= 6m - 21mn$$

Solving equations with brackets

To solve an equation with one pair of brackets, decide whether it will be easier to expand the brackets first or to divide by the number in front of the brackets.

Solve for k. $3(k - 5) = 6$

$3 \times k + 3 \times -5 = 6$
$3k - 15 = 6$
$+15 \quad +15$
$3k = 21$
$\div 3 \quad \div 3$
$k = 7$

$3(k - 5) = 6$
$\div 3 \quad \div 3$
$k - 5 = 2$
$+5 \quad +5$
$k = 7$

To check, substitute $k = 7$ into $3(k - 5) = 6$

$3(7 - 5) = 3 \times 2 = 6$ ✔

If an equation has more than one pair of brackets, expand all the brackets and simplify before solving.

Solve $3(2j - 4) + 2(j + 3) = 26$

First expand both brackets.

$3(2j - 4)$
$= (3 \times 2j) + (3 \times -4)$
$= 6j - 12$

$2(j + 3)$
$= (2 \times j) + (2 \times 3)$
$= 2j + 6$

$6j - 12 + 2j + 6 = 26$
$8j - 6 = 26$
$+6 \quad +6$
$8j = 32$
$\div 8 \quad \div 8$
$j = 4$

RETRIEVE

4 Solving equations with brackets

Expanding brackets using the distributive law

1 Expand each expression and simplify where possible by combining like terms.

a) $3(2k - 4) + 5k$

b) $k(2m - 4)$

c) $4(2x + 3y - 5)$

Solving equations with brackets

2 Solve the following equations.

a) $3(k - 5) = 12$

$k =$ ....

b) $2x + 3(4 - x) = 10$

$x =$ ....

3 Solve the following equations.

a) $2(x + 3) + 4(x - 5) = 10$

$x =$ ....

b) $5(y - 2) = 3(y - 2) + 2$

$y =$ ....

ORGANISE

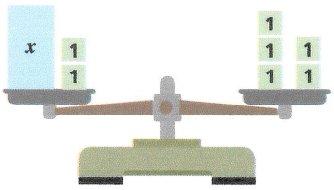
4 Solving one-step equations

Equations and inverse operations

An expression is a collection of **terms** that can be **variables** or **constants** without an equals symbol.

An **equation** consists of one or more **expressions**. Both sides of the equation are equal in value (as shown by the = symbol).

A set of **balance scales** can help to visualise equations. This set of scales represents the equation $x + 2 = 5$.



To solve equations, you need to use **inverse operations** (these 'undo' other operations).

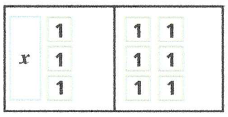
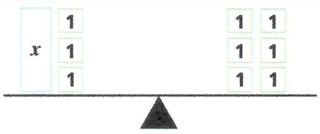
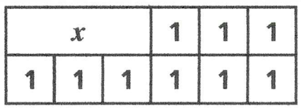
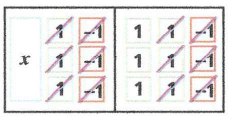
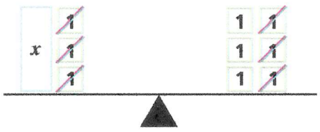
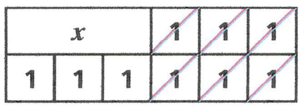
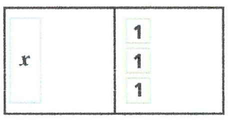
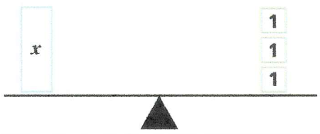
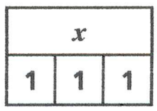
Inverse operations		
Addition +	⟺	Subtraction −
Multiplication ×	⟺	Division ÷
Powers (e.g. x^2, x^3)	⟺	Roots (e.g. $\sqrt{x}$, $\sqrt[3]{x}$)

Using visual aids to solve equations

To '**solve** for x' means to find the value of x. The x term must be **isolated** on one side of the equation. The aim is to get the variables to one side of the equation and the constants to the other.

This equation mat, scales and bar model all show the equation $x + 3 = 6$.

Whatever you do to one side of the equation, you must do to the other to keep it balanced.

Algebraic steps	Equation mat	Scales	Bar model
Lay out the equation. $x + 3 = 6$			
Use inverse operations to isolate x. $x + 3 - 3 = 6 - 3$ Subtract 3 from both sides	Place three −1 tiles on both sides. The +1 and −1 tiles 'cancel out' because they add up to 0. 	Remove 3 ones tiles from both sides. 	'Cancel out' the parts that are the same on both bars. 
Simplify $x = 3$			

Solving without visual aids

To solve an equation without visual aids, use inverses and carry out the same operation on both sides to keep it balanced like a set of scales.

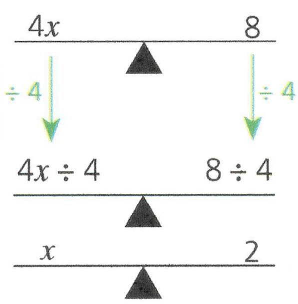
$4x = 8$

The inverse of × 4 is ÷ 4, so divide both sides by 4.

$4x \div 4 = 8 \div 4$

$x = 2$

To check: $4 \times 2 = 8$ ✔



RETRIEVE

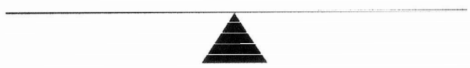
4 Solving one-step equations

Equations and inverse operations

1 Show each equation on a set of balance scales using algebra tiles.

a) $2x + 4 = 8$

b) $3x + 2 = 11$



Using visual aids to solve equations

2 Show $x + 6 = 8$ using algebra tiles and find the value of x.

$x =$ ....

Solving without visual aids

3 Solve the following equations for x.

a) $x + 9 = 15$

$x =$ ....

b) $x \div 5 = 3$

$x =$ ....

c) $12x = 96$

$x =$ ....

ORGANISE

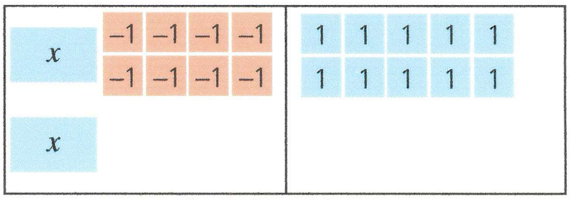
4 Solving two-step equations

Using algebra tiles

Algebra tiles can help to visualise how to solve two-step equations.

> Use inverse operations to combine like terms so that all the variables are on one side of the equation and the constants on the other.

Consider $2x - 8 = 10$. There are 2 x tiles and 8 negative one tiles on the left-hand side. There are 10 positive one tiles on the right-hand side.

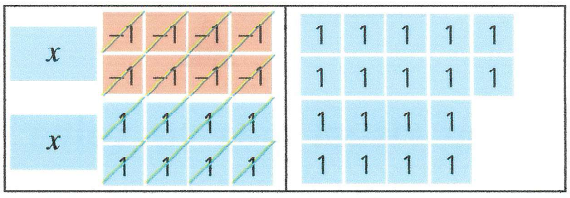


$2x - 8 = 10$

To get x on its own on one side of the equation, use inverse operations and add 8 to both sides.

Simplify by combining the ones tiles.

The 8 positive tiles 'cancel out' the 8 negative ones.

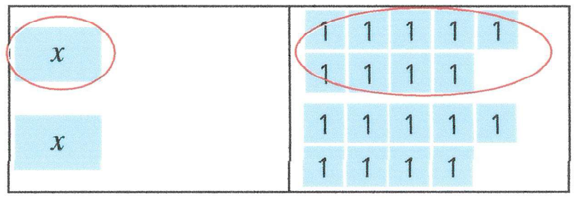


$2x - 8 + 8 = 10 + 8$

$2x = 18$

Now divide each side into two equal groups to find the value of $1x$.

Two x tiles equal 18, so one x tile equals 9.



$2x \div 2 = 18 \div 2$

$x = 9$

To check: $2 \times 9 - 8 = 18 - 8 = 10$ ✔

Solving without visual aids

To solve two-step equations without visual aids, use inverse operations and write down each step. Carry out the same operation on both sides. To solve $2x - 8 = 10$ without algebra tiles, you can first divide by 2 (since all terms are divisible by 2) or you can add 8 to both sides.

$2x - 8 = 10$ $+8 \quad +8$	Add 8 to both sides.	$2x - 8 = 10$ $\div 2 \quad \div 2$	Divide both sides by 2.
$2x = 18$ $\div 2 \quad \div 2$	Divide both sides by 2.	$x - 4 = 5$ $+4 \quad +4$	Add 4 to both sides.
$x = 9$		$x = 9$	

To solve $5x - 9 = 16$, it is not sensible to divide each term before solving because there is not a common factor between 5, 9 and 16.

$5x - 9 = 16$

$+9 \quad +9$ — Add 9 to both sides.

$5x = 25$ — Simplify.

$\div 5 \quad \div 5$ — Divide both sides by 5.

$x = 5$

To check: $5 \times 5 - 9 = 25 - 9 = 16$ ✔

a) **Find the value of n in $3n - 5 = 31$**

$3n - 5 = 31$

$+5 \quad +5$ — Add 5 to both sides.

$3n = 36$

$\div 3 \quad \div 3$ — Divide both sides by 3.

$n = 12$

To check: $3 \times 12 - 5 = 36 - 5 = 31$ ✔

b) **Find the value of a in $\frac{a}{4} - 3 = 2$**

> Remember that a fraction is another way of writing division, so $\frac{a}{4}$ means $a \div 4$.

$\frac{a}{4} - 3 = 2$

$+3 \quad +3$ — Add 3 to both sides.

$\frac{a}{4} = 5$

$\frac{a}{4} \times 4 = 5 \times 4$ — Multiply both sides by 4.

$a = 20$

To check: $\frac{20}{4} - 3 = (20 \div 4) - 3$

$= 5 - 3 = 2$ ✔

RETRIEVE

4 Solving two-step equations

Using algebra tiles

1 Show the equation $2x - 5 = 9$ using algebra tiles and solve for x.

If you don't have algebra tiles to hand, you can instead cut up bits of paper and label them.

$x =$ ....

Solving without visual aids

2 Solve each of the following equations. Show your working.

a) $6x + 10 = 28$

$x =$ ....

b) $\frac{x}{2} + 3 = 6$

$x =$ ....

c) $5x - 4 = 41$

$x =$ ....

d) $\frac{x}{6} + 8 = 14$

$x =$ ....

ORGANISE 3

Solving multi-step equations 1

Solving one- and two-step equations

Equations are solved using **inverse operations**.

The **one-step equation** $x + 2 = 5$ can be solved as follows:

$x + 2 = 5$
$-2 \quad -2$
$x = 3$

Inverse operations		
Addition +	⇔	Subtraction –
Multiplication ×	⇔	Division ÷
Powers (e.g. x^2, x^3)	⇔	Roots (e.g. $\sqrt{x}$, $\sqrt[3]{x}$)

The process is the same for solving **two-step equations**, except you must first isolate the unknown term (usually the variable x) on one side of the equation. It does not matter which side the unknown term is on.

Solve the equation $3x - 8 = 10$

$3x - 8 = 10$
$+8 \quad +8$
Use inverse operations to isolate x. The inverse of – 8 is + 8, so add 8 to both sides.

$3x = 18$
$\div 3 \quad \div 3$
$x = 6$
Now use inverse operations to find the value of x. $3x$ means 3 times x, and the inverse of multiplication is division, so divide both sides by 3.

When solving an equation, carry out the same operation to both sides at each step.

Solving equations with variables on both sides

To solve equations with variables on both sides, isolate the unknown term to one side.

Solve $2x + 8 = 4x - 2$

Subtracting $2x$ from both sides leaves a positive x term, whereas subtracting $4x$ from both sides would leave a negative x term. So it is easier to subtract $2x$.

$2x + 8 = 4x - 2$
$-2x \quad -2x$
Subtract $2x$ from both sides.

$8 = 2x - 2$
$+2 \quad +2$
Use inverse operations to isolate x.

$10 = 2x$
$\div 2 \quad \div 2$
To find the value of x, divide both sides by 2.

$5 = x$
$x = 5$
The answer is usually written with x on the left-hand side.

Solve $7y - 5 = 5y + 3$

$7y - 5 = 5y + 3$
$-5y \quad -5y$
Subtract $5y$ from both sides.

$2y - 5 = 3$
$+5 \quad +5$
Add 5 to both sides.

$2y = 8$
$\div 2 \quad \div 2$
Divide both sides by 2.

$y = 4$

Solving equations with brackets

When an equation has brackets, first expand them, then simplify and proceed to solve the equation as normal.

Solve $2(3x + 8) - 3(x + 4) = 10$

$2(3x + 8) - 3(x + 4) = 10$
First expand $2(3x + 8) = 6x + 16$ and $-3(x + 4) = -3x - 12$

$6x + 16 - 3x - 12 = 10$
Combine like terms.

$3x + 4 = 10$
$-4 \quad -4$
The inverse of + 4 is – 4, so subtract 4 from both sides.

$3x = 6$
$\div 3 \quad \div 3$
$3x$ means $3 \times x$, so divide both sides by 3.

$x = 2$

RETRIEVE

3 Solving multi-step equations 1

Solving one- and two-step equations

1 Solve each equation.

a) $7x + 9 = 51$

$x =$ ....

b) $6x - 9 = 21$

$x =$ ....

Solving equations with variables on both sides

2 Solve each equation.

a) $2x + 8 = 4x - 4$

$x =$ ....

b) $6x - 1 = 15 - 2x$

$x =$ ....

Solving equations with brackets

3 Solve each equation.

a) $3(x - 5) + 2(9 - x) = 10$

$x =$ ....

b) $6x + 7(2x - 3) = 19$

$x =$ ....

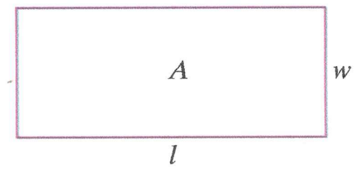
ORGANISE

3 Using and writing formulae

Writing formulae

A **formula** is a rule that shows how two or more **variables** are linked. The variable that is being worked out is called the **subject**.

$A = l \times w$ is a formula. It shows how the variables of length, width and area of a rectangle are related. A is the subject.



For example, a plumber charges a £100 call-out fee plus an additional £25 for every hour worked:

- In words, to calculate the cost, multiply £25 by the number of hours worked and add that to the call-out fee of £100.
- Writing the formula algebraically, state the variables. Let C = the total cost the plumber charges and h = the number of hours worked. Then the formula is $C = 100 + 25h$

Cost (the subject)
Call-out fee
Price per hour
Number of hours (the variable)

A formula shows how two or more variables are related. When writing a formula algebraically, make sure you state what each variable represents.

Using formulae

To use a formula, substitute in the given values of the known variables and find the unknown variable.

A taxi company charges a £5 flat rate plus £1.25 per mile.

a) Write the formula in words.

Multiply the cost per mile, £1.25, by the number of miles travelled and add the product to the flat rate of £5.

b) Write the formula algebraically.

$C = 5 + 1.25m$, where C = the total cost and m = the number of miles travelled

c) Find the cost of taking a taxi for 8 miles.

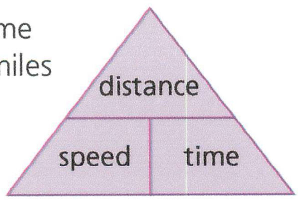
$C = 5 + 1.25m$

$C = 5 + (1.25 \times 8)$

$C = £15$

To find the cost for travelling 8 miles, substitute $m = 8$.

Speed is the distance travelled per unit of time (e.g. 50 miles per hour means travelling 50 miles in one hour). This relationship can be shown using a formula triangle.



To use the triangle, cover the subject and multiply or divide the other variables as appropriate.

The triangle shows three formulae:
Speed = distance ÷ time
Time = distance ÷ speed
Distance = speed × time

A plane travels at an average speed of 900 km/h for 8.5 hours. How far has it travelled?

Use the formula triangle, $D = S \times T$, where D = Distance, S = Speed and T = Time.

$D = S \times T$

$D = 900 \times 8.5$

$D = 7650\text{km}$

Substitute $T = 8.5\text{h}$ and $S = 900\text{km/h}$.

The formula $C = \frac{5(F-32)}{9}$ can be used to convert between degrees Celsius (°C) and degrees Fahrenheit (°F).

Find the temperature in Celsius when it is 85°F, giving the answer to 1 decimal place.

Substitute $F = 85$ into the formula $C = \frac{5(F-32)}{9}$

$C = \frac{5(85-32)}{9}$ — First work out the brackets, 85 − 32 = 53

$= \frac{5 \times 53}{9}$ — A number written outside brackets means to multiply. 5 × 53 = 265

$= \frac{265}{9}$ — A fraction is another way of writing division, so divide. $265 \div 9 = 29.\dot{4}$

$= 29.4°\text{C}$ (to 1 d.p.)

RETRIEVE

3 Using and writing formulae

Writing formulae

1 Write a formula in words and algebraically for each situation.

a) Finding the perimeter of a regular polygon.

In words: ....

Algebraically: ....

b) 

In words: ....

Algebraically: ....

Using formulae

2 Use the formulae from question 1 to find the following.

a) The perimeter of a regular decagon (10 sides) with side lengths of 1.3 cm

.... cm

b) The cost of a party with 25 guests

£ ....

3 A train travels 630 km at a speed of 280 km/h.

How many hours does the journey take?

....

ORGANISE

3 Rearranging formulae

The subject of a formula

The subject of a formula is the variable that is isolated, usually to the left of the equals sign.

Identify the subject in each formula.

a) $A = \frac{b \times h}{2}$

A is the subject. The formula shows how the base and height of a triangle are related to the area.

b) $P = 2w + 2l$

P is the subject. The formula shows how the length and width of a parallelogram are related to the perimeter.

It is sometimes useful to rearrange the formula to **change the subject** to a different variable.

Suppose you know the area and height of a triangle and want to find the base. It would be easier if b was the subject of the formula.

How to rearrange formulae

Rearranging formulae is just like solving equations. Use inverse operations to 'undo' the formula so that a different variable is isolated.

Inverse operations		
Addition +	⇔	Subtraction −
Multiplication ×	⇔	Division ÷
Powers (e.g. x^2, x^3)	⇔	Roots (e.g. $\sqrt{x}$, $\sqrt[3]{x}$)

You can think of formulae as function machines and work backwards to rearrange the subject. To draw a function machine, think about which variable is the input and which variable is the output. To find the input from the output, work backwards using inverse operations.

Function machine:

input → × 3 → + 4 → output

Reverse function machine:

input ← ÷ 3 ← − 4 ← output

Think of rearranging formulae as the same as solving equations. Get the new subject of the formula on its own by using inverse operations.

The formula for the area of a triangle is $A = \frac{bh}{2}$. To rearrange the formula to make the base, b, the subject, think about the operations that are required and apply the inverses.

base → × height → ÷ 2 → area

base ← ÷ height ← × 2 ← area

To show this algebraically, think about undoing each operation to isolate b.

$A = \frac{bh}{2}$

$A \times 2 = \frac{bh}{2} \times 2$ — Multiply both sides by 2 to 'undo' the fraction.

$2A = bh$ — Simplify.

$\div h \quad \div h$ — Divide both sides by h to solve for b.

$2A \div h = b$

$b = \frac{2A}{h}$ — Rewrite the formula to have the b term (the new subject) on the left-hand side.

Rearrange the formula $V = \pi r^2 h$ to make r the subject.

Using a function machine:

r → squared → × π → × h → V

Reverse function machine:

r ← square root ← ÷ π ← ÷ h ← V

Algebraically: $V = \pi r^2 h$

$\div \pi h \quad \div \pi h$ — Divide both sides by πh to isolate the r term.

$\frac{V}{\pi h} = r^2$

$\sqrt{\frac{V}{\pi h}} = \sqrt{r^2}$ — Take the square root of both sides to 'undo' r^2.

$r = \sqrt{\frac{V}{\pi h}}$ — Rewrite the formula to have the r term on the left-hand side.

RETRIEVE

3 Rearranging formulae

The subject of a formula

1 Write down the subject of each formula.

a) $P = sl$

b) $A = \pi r^2$

c) $C = 30h + 45$

How to rearrange formulae

2 You are given the formula $P = sl$, where P is the perimeter of a regular polygon, s is the number of sides and l is the side length.

a) Rearrange the formula to make l the subject.

b) Find the length of each side of a 12-sided polygon with perimeter of 108 cm.

3 You are given the formula $A = \pi r^2$, where A is the area of a circle and r is the radius.

a) Rearrange the formula to make r the subject.

b) Find the radius of a circle with area 225π cm^2.

4 You are given the formula $C = 30h + 45$, where C is the charge in £ for a DJ and h is the number of hours worked by the DJ.

a) Rearrange the formula to make h the subject.

b) Find the number of hours worked by the DJ if the charge was £135.