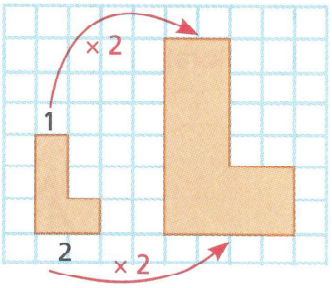
8 Enlargement

Enlarging shapes

An enlargement multiplies each length on a shape by the same **scale factor**.

The enlarged shape is called the image. It is the same shape as the original shape, but it is not the same size.

Each length in the smaller shape is multiplied by 2 to create the larger shape. The smaller shape has been enlarged by scale factor 2. You could draw this enlargement anywhere on the grid.

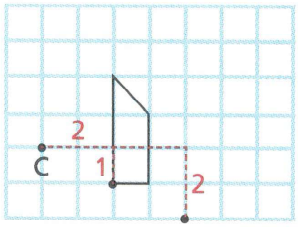


Enlarging about a centre

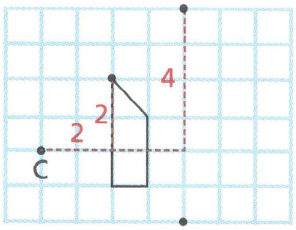
To enlarge about a centre, multiply the distance from the centre to each point on the shape by the scale factor.

Enlarging about a centre tells you where to draw the enlarged shape.

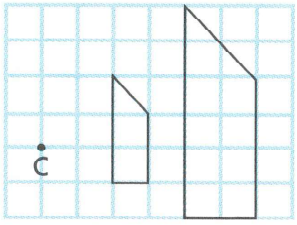
To enlarge this shape by scale factor 2, centre C, work out the distance from C to each vertex (corner) in turn.



From C to this vertex is 2 across, 1 down. Multiply each distance by 2. Draw the image of the point 4 across and 2 down from C.



From C to this vertex is 2 across, 2 up. Multiply each distance by 2. Draw the image of the point 4 across and 4 up from C.

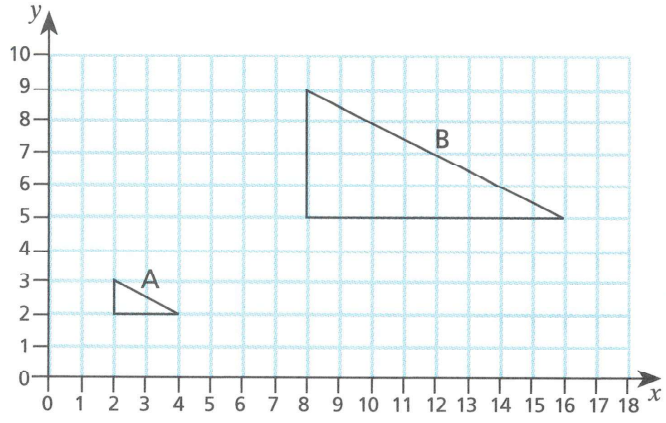


Work out the position of the other vertices in the same way. Join them with straight lines. Check that all the sides in the enlarged shape are twice the length of those in the original shape.

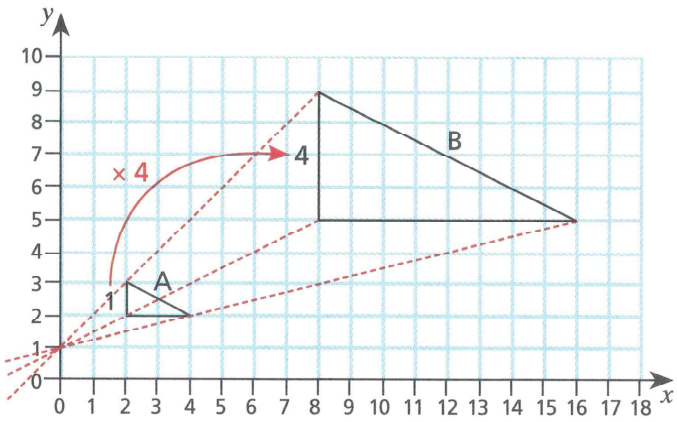
Describing enlargements

To describe an enlargement, you need to state the **scale factor** and the **centre of the enlargement**.

Describe the enlargement that takes shape A to shape B.



Find the scale factor of enlargement by comparing side lengths on the shapes.



To find the centre of the enlargement, draw straight lines between corresponding vertices on the two shapes. The centre of enlargement is where these lines cross.

Enlargement by scale factor 4, centre (0, 1)

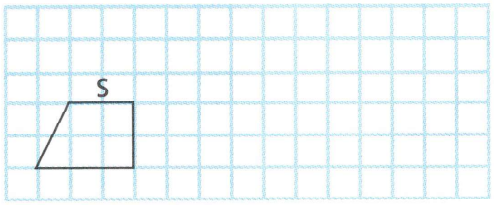
RETRIEVE

8 Enlargement

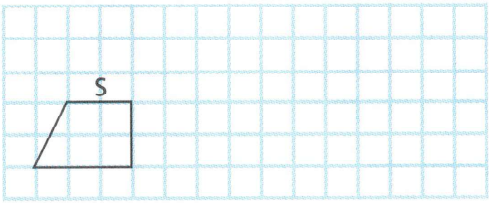
Enlarging shapes

1 Enlarge shape S:

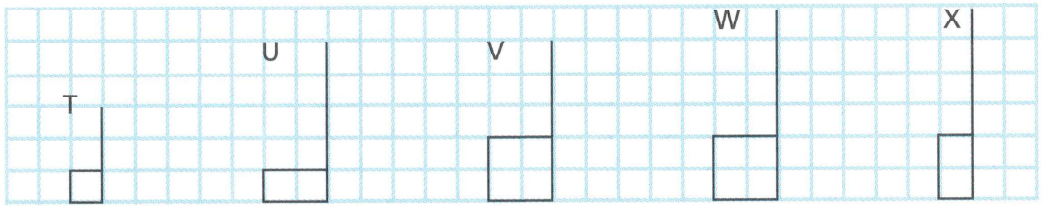
a) by scale factor 2



b) by scale factor 3

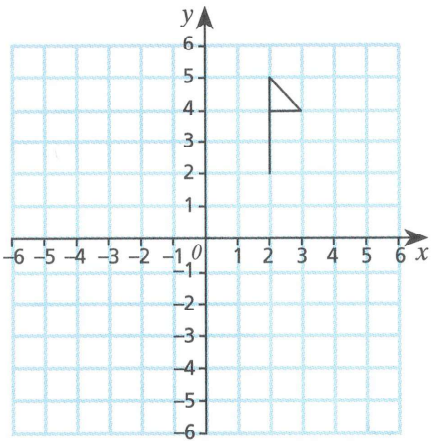


2 Which shape is an enlargement of shape T by scale factor 2? Circle your answer.

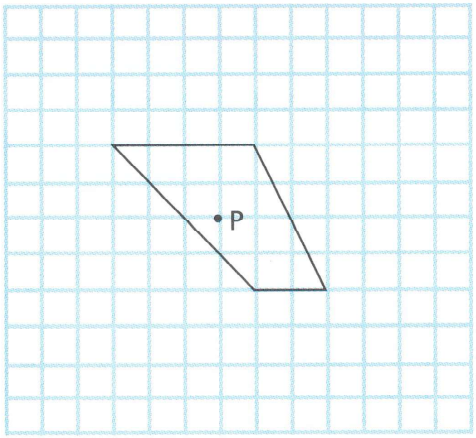


Enlarging about a centre

3 Enlarge this shape by scale factor 2 about centre (1, 6).

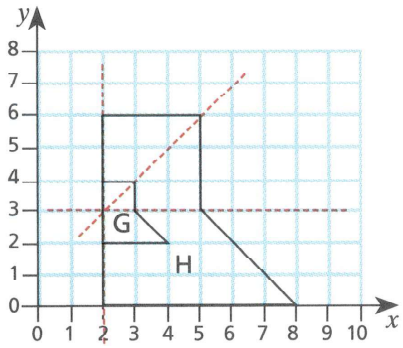


4 Enlarge this shape by scale factor 2 about centre P.



Describing enlargements

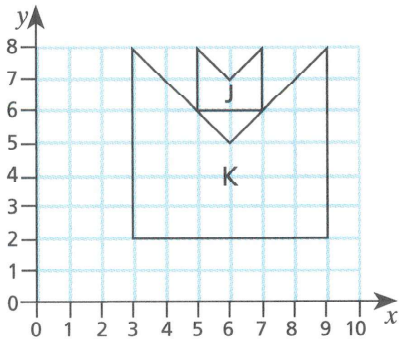
5 Describe the enlargement that takes shape G to shape H.



An enlargement of

scale factor ....., centre .....

6 Describe the enlargement that takes shape J to shape K.



.....

.....

6 Adding fractions

Adding fractions with the same denominator

To add fractions with the same denominator, simply add the numerators. The answer may be greater than 1.

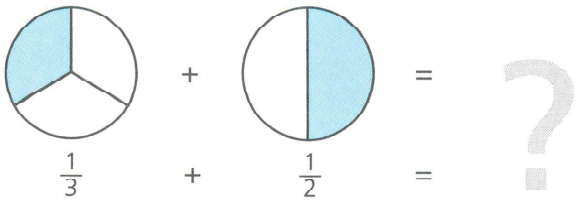
$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$

$\frac{5}{7} + \frac{4}{7} = \frac{5+4}{7} = \frac{9}{7} = 1\frac{2}{7}$

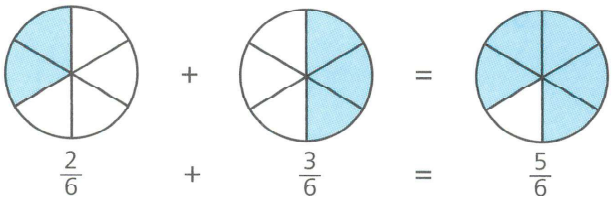
Adding fractions with different denominators

To add fractions, the denominators must first be the same.

You can't add $\frac{1}{3} + \frac{1}{2}$ directly.



However, using a common denominator of 6:



$\frac{3}{4} + \frac{6}{18} = ?$

A common multiple of 4 and 18 is 36.

$\frac{3}{4} \xrightarrow{\times 9} \frac{27}{36}$ $\qquad$ $\frac{6}{18} \xrightarrow{\times 2} \frac{12}{36}$

$\frac{3}{4} + \frac{6}{18} = \frac{27}{36} + \frac{12}{36} = \frac{27+12}{36}$

$= \frac{39}{36} = \frac{13}{12} = 1\frac{1}{12}$

Find a common denominator and use equivalent fractions to add the fractions.

Adding mixed numbers

Method 1: Partition the mixed numbers into their integer and fraction components, then add them.

Method 2: Convert the mixed numbers to improper fractions, then add them, and finally convert the answer back to a mixed number if needed.

Just as when adding proper fractions, the fractions must have a common denominator.

If the answer has an improper fraction in it as well, convert that to a mixed number and add it to the integer component.

$1\frac{3}{5} + 2\frac{1}{8} = ?$

Method 1:

$1 + 2 + \frac{3}{5} + \frac{1}{8}$ First partition the integers and fractions.

$1 + 2 = 3$ Add the integers.

$\frac{3}{5} \xrightarrow{\times 8} \frac{24}{40}$ $\qquad$ $\frac{1}{8} \xrightarrow{\times 5} \frac{5}{40}$

Now add the fractions. Find a common denominator. 5 and 8 have a common multiple of 40.

$\frac{24}{40} + \frac{5}{40} = \frac{29}{40}$

Therefore, $1\frac{3}{5} + 2\frac{1}{8} = 3\frac{29}{40}$

Method 2:

Convert the mixed numbers to improper fractions.

$1\frac{3}{5} = \frac{(5 \times 1) + 3}{5} = \frac{8}{5}$ $\qquad$ $2\frac{1}{8} = \frac{(8 \times 2) + 1}{8} = \frac{17}{8}$

$\frac{8}{5} \xrightarrow{\times 8} \frac{64}{40}$ $\qquad$ $\frac{17}{8} \xrightarrow{\times 5} \frac{85}{40}$

Find a common denominator.

$\frac{64}{40} + \frac{85}{40} = \frac{149}{40}$

$149 \div 40 = 3$ r29, so $\frac{149}{40} = 3\frac{29}{40}$

Convert the improper fraction to a mixed number.

$1\frac{3}{5} + 2\frac{1}{8} = 3\frac{29}{40}$

RETRIEVE

6 Adding fractions

Adding fractions with the same denominator

1 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a) $\frac{4}{8} + \frac{3}{8}$

b) $\frac{12}{20} + \frac{15}{20}$

Adding fractions with different denominators

2 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a) $\frac{3}{12} + \frac{5}{8}$

b) $\frac{7}{9} + \frac{3}{4}$

Adding mixed numbers

3 Add these fractions. Give your answer as a mixed number when the answer is greater than 1.

a) $7\frac{2}{5} + 3\frac{1}{8}$

b) $5\frac{4}{5} + 1\frac{3}{8}$

6 Subtracting fractions

Subtracting fractions with the same denominator

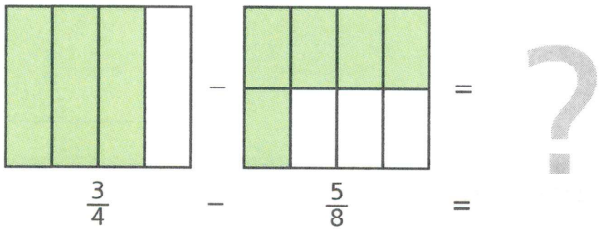
Subtracting fractions is the same process as adding fractions, just using the operation of subtraction rather than addition.

$\frac{8}{12} - \frac{5}{12} = \frac{8-5}{12} = \frac{3}{12} = \frac{1}{4}$

Subtracting fractions with different denominators

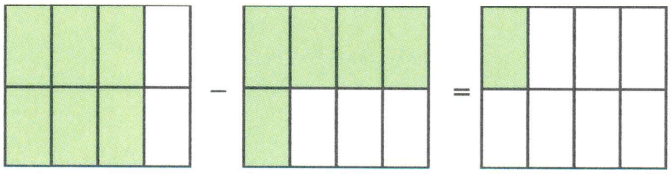
To subtract fractions, the denominators must first be the same. If they are different, find a common denominator and use equivalent fractions.

You can't subtract $\frac{3}{4} - \frac{5}{8}$ directly.



$\frac{3}{4} - \frac{5}{8} =$

However, using a common denominator of 8:



$\frac{6}{8} - \frac{5}{8} = \frac{1}{8}$

Work out $\frac{5}{8} - \frac{3}{12}$

$\frac{5}{8} \overset{\times 3}{=} \frac{15}{24}$ $\quad \frac{3}{12} \overset{\times 2}{=} \frac{6}{24}$

A common multiple of 8 and 12 is 24.

$\frac{5}{8} - \frac{3}{12} = \frac{15}{24} - \frac{6}{24}$

$= \frac{15-6}{24}$

$= \frac{9}{24} = \frac{3}{8}$

Find a common denominator and use equivalent fractions to subtract fractions.

Subtracting mixed numbers

Method 1: Partition the mixed numbers into their integer and fraction components, then subtract them.

Method 2: Convert the mixed numbers to improper fractions, then subtract them, and finally convert the answer back to a mixed number if needed.

You may find one method easier to use than the other. Learning how to use the fraction buttons on a calculator will help.

Work out $8\frac{1}{2} - 4\frac{5}{8}$

Method 1:

$8\frac{1}{2} - 4\frac{5}{8} = 8 + \frac{1}{2} - 4 - \frac{5}{8}$ Partition the mixed numbers.

Keep track of the negative signs. Don't forget you are subtracting the integer and the fraction!

$8 - 4 = 4$ Subtract the integers.

$\frac{1}{2} = \frac{4}{8}$

$\frac{4}{8} - \frac{5}{8} = \frac{4-5}{8} = -\frac{1}{8}$ Subtract the fractions. Find a common denominator, 8.

$8\frac{1}{2} - 4\frac{5}{8} = 4 - \frac{1}{8}$ This is where it gets tricky.

The answer is $\frac{1}{8}$ less than 4, which is $3\frac{7}{8}$

Method 2:

$8\frac{1}{2} = 8\frac{4}{8}$ Find a common denominator, 8.

$8\frac{4}{8} = \frac{(8 \times 8) + 4}{8} = \frac{68}{8}$

$4\frac{5}{8} = \frac{(8 \times 4) + 5}{8} = \frac{37}{8}$

Convert to improper fractions and subtract.

$\frac{68}{8} - \frac{37}{8} = \frac{31}{8} = 3\frac{7}{8}$

RETRIEVE

6 Subtracting fractions

Subtracting fractions with the same denominator

1 Subtract these fractions.

a) $\frac{5}{8} - \frac{2}{8}$

b) $\frac{13}{15} - \frac{8}{15}$

Subtracting fractions with different denominators

2 Subtract these fractions.

a) $\frac{6}{7} - \frac{3}{14}$

b) $\frac{5}{7} - \frac{4}{9}$

Subtracting mixed numbers

3 Subtract these fractions. Give your answer as a mixed number when the answer is greater than 1.

a) $4\frac{2}{3} - 2\frac{1}{8}$

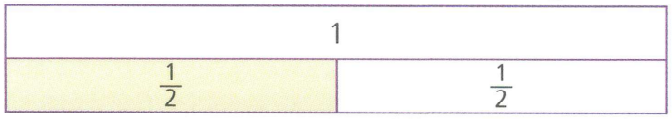
b) $2\frac{2}{5} - 1\frac{5}{7}$

6 Multiplying fractions

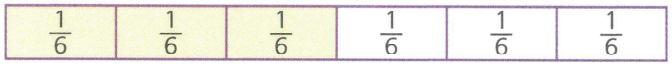
Multiplying fractions by fractions

Multiplying by a fraction means finding a fraction of an amount. Multiplying two fractions means finding a fraction of the fraction.

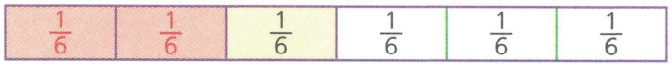
$\frac{2}{3} \times \frac{1}{2}$ means finding $\frac{2}{3}$ of $\frac{1}{2}$. This can be shown using a bar model. Here is a bar showing $\frac{1}{2}$:



Now divide both parts of the bar into thirds. The bar now has 6 parts with half of the bar shaded.



Then find $\frac{2}{3}$ of the shaded part.



$\frac{2}{3}$ of $\frac{1}{2}$ of the bar is $\frac{2}{6}$ of the bar, or $\frac{1}{3}$

To multiply fractions without a bar model:

- multiply the numerator by the numerator
- multiply the denominator by the denominator
- simplify the fraction.

Work out $\frac{2}{3} \times \frac{6}{8}$

$\frac{2}{3} \times \frac{6}{8} = \frac{2 \times 6}{3 \times 8} = \frac{12}{24} = \frac{1}{2}$

To multiply mixed numbers, first convert them to improper fractions.

Work out $2\frac{3}{4} \times 1\frac{2}{5}$

$2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{11}{4}$ $\quad 1\frac{2}{5} = \frac{(5 \times 1) + 2}{5} = \frac{7}{5}$

$\frac{11}{4} \times \frac{7}{5} = \frac{11 \times 7}{4 \times 5}$

$= \frac{77}{20} = 3\frac{17}{20}$

Then multiply the improper fractions and simplify the answer.

To do these calculations on a calculator, find the mixed number button and enter the calculation.

Multiply the numerator by the numerator and the denominator by the denominator.

Multiplying fractions by integers

To multiply a fraction by an integer, remember that an integer can be written as a fraction by writing the integer as the numerator and writing the denominator as 1.

Then to multiply the fraction and integer, multiply the numerators and the denominators as before.

$5 \times \frac{2}{3} = \frac{5}{1} \times \frac{2}{3}$ $\quad 5 = \frac{5}{1}$

$= \frac{5 \times 2}{1 \times 3} = \frac{10}{3} = 3\frac{1}{3}$

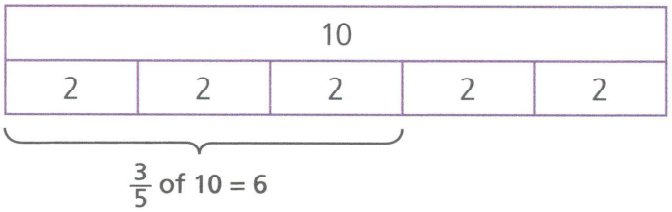
The step of writing the integer as a fraction can be left out. Multiply the integer by the numerator and keep the denominator the same (because the denominator was being multiplied by 1).

Multiply the integer by the numerator and keep the denominator the same.

Finding a fraction of an amount

Finding a fraction of an amount means to multiply that amount by a fraction.

$\frac{3}{5}$ of £10 can be shown using a bar model.



Find $\frac{4}{5}$ of £20

£20 ÷ 5 = £4 so $\frac{1}{5}$ of £20 is £4

To find $\frac{4}{5}$, first find $\frac{1}{5}$

Then $\frac{4}{5}$ of £20 is 4 × £4 = £16

Alternatively, multiply the amount by the numerator and divide by the denominator.

$\frac{4}{5} \times 20 = \frac{(4 \times 20)}{5} = \frac{80}{5}$

80 ÷ 5 = 16, so $\frac{4}{5}$ of £20 is £16

RETRIEVE

6 Multiplying fractions

Multiplying fractions by fractions

1 Multiply these fractions. Simplify your answers.

a) $\frac{7}{9} \times \frac{5}{8}$

.....

b) $\frac{2}{5} \times \frac{3}{4}$

.....

c) $1\frac{5}{6} \times 2\frac{2}{5}$

.....

Multiplying fractions by integers

2 Multiply these fractions and integers. Simplify your answers.

a) $3 \times \frac{5}{8}$

.....

b) $9 \times \frac{3}{4}$

.....

c) $8 \times \frac{2}{5}$

.....

Finding a fraction of an amount

3 Find these fractions of amounts.

a) $\frac{4}{5}$ of £200

£.....

b) $\frac{2}{7}$ of 49 sweets

..... sweets

c) $\frac{5}{9}$ of 54 cm

..... cm

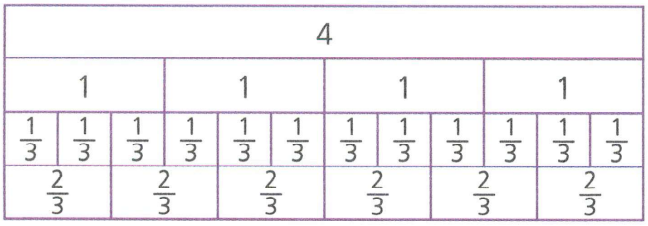
6 Dividing fractions

Dividing an integer by a fraction

Division is another way of asking how many times one quantity fits into another. Dividing an integer by a fraction is asking how many times that fraction fits into the integer.

Work out $4 \div \frac{2}{3}$

To find $4 \div \frac{2}{3}$, a bar with a value of 4 can be divided into parts representing $\frac{2}{3}$



There are 6 sections showing $\frac{2}{3}$

Therefore, $4 \div \frac{2}{3} = 6$

Without a bar model, you can carry out division calculations involving fractions by using the **reciprocal method**. This involves keeping the first number the same, using the reciprocal of the second number, and changing the division to multiplication.

Keep the first number the same
Flip the second number
Change ÷ to ×

To find the reciprocal of a fraction, the numerator becomes the denominator, and the denominator becomes the numerator. Think of the reciprocal of a fraction as the fraction flipped upside down.

Work out $6 \div \frac{2}{3}$

$$6 \div \frac{2}{3} = 6 \times \frac{3}{2}$$

Keep the first value the same — Change ÷ to × — Flip the second value

$$= \frac{6 \times 3}{2} = \frac{18}{2} = 9$$

Dividing a fraction by a fraction

Dividing a fraction by a fraction means asking how many times one fraction fits into another.

To divide a fraction by a fraction, use the KFC method.

$\frac{3}{4} \div \frac{1}{2}$ means how many halves are in three quarters.

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$$

Keep the first value the same — Change ÷ to × — Flip the second value

$$= \frac{3 \times 2}{4 \times 1}$$

$$= \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

Work out $\frac{2}{5} \div \frac{7}{9}$

$$\frac{2}{5} \div \frac{7}{9} = \frac{2}{5} \times \frac{9}{7}$$

$$= \frac{2 \times 9}{5 \times 7}$$

$$= \frac{18}{35}$$

You can remember the reciprocal method by KFC: Keep, Flip, Change

Dividing with mixed numbers

If the calculation involves mixed numbers, first convert the mixed number to an improper fraction. Then use the reciprocal method to divide.

Work out $2\frac{1}{3} \div \frac{5}{9}$

$2\frac{1}{3} = \frac{(3 \times 2) + 1}{3} = \frac{7}{3}$ — Convert $2\frac{1}{3}$ to an improper fraction.

$\frac{7}{3} \div \frac{5}{9} = \frac{7}{3} \times \frac{9}{5}$ — Use the reciprocal method.

$$= \frac{7 \times 9}{3 \times 5} = \frac{63}{15}$$

$\frac{63}{15} = 4\frac{3}{15} = 4\frac{1}{5}$ — Simplify the answer if possible.

RETRIEVE

6 Dividing fractions

Dividing an integer by a fraction

1 Divide these integers and fractions.

a) $9 \div \frac{2}{3}$

b) $6 \div \frac{5}{6}$

Dividing a fraction by a fraction

2 Divide these fractions.

a) $\frac{1}{6} \div \frac{3}{8}$

b) $\frac{1}{9} \div \frac{9}{4}$

c) $\frac{1}{2} \div \frac{2}{3}$

Dividing with mixed numbers

3 Divide these fractions and mixed numbers.

a) $3\frac{2}{7} \div \frac{1}{2}$

b) $5\frac{2}{3} \div \frac{1}{3}$